Introduction to general relativity

Gravity is just geometry in disguise

Intuitions behind general relativity

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SR works

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Equivalence principle

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Weak equivalence principle

$$\vec{F} = m_i \vec{a}$$

inertial mass

$$\vec{F} = -m_g \vec{\nabla} \Phi$$

gravitational mass

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All bodies in free fall undergo the same acceleration **WEP 2**

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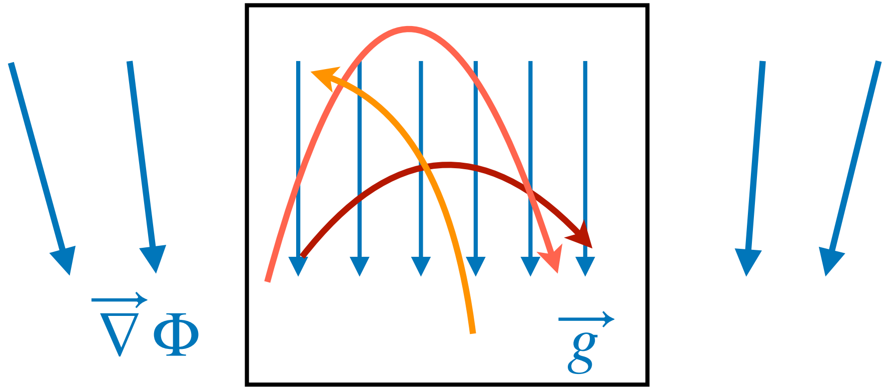
All bodies in free fall undergo the same acceleration **WEP 2**

Idea: The set of trajectories of free falling bodies must be a fundamental property built into the spacetime itself, rather than an effect of a force acting on a particular body

Equivalence principle

Gravity within a small box

$$\vec{\nabla}\Phi \approx -\vec{g} = \text{const}$$



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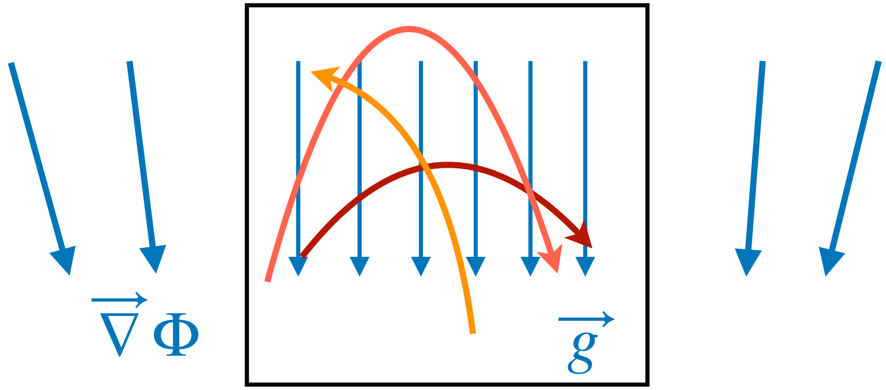

Free fall

$$\ddot{\vec{x}} = \vec{g}$$

$$\vec{x}(t) = \vec{x}_0 + \vec{v}\,t + \frac{1}{2}\vec{g}\,t^2$$

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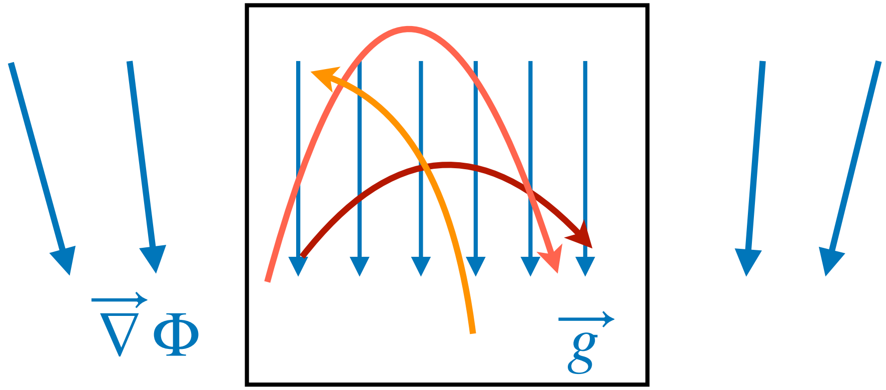

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Relative motion between free falling bodies

$$\vec{x}(t) - \vec{x}'(t) = (\vec{x}_0 - \vec{x}_0') + (\vec{v} - \vec{v}')\,t$$

Moving with constant velocity wrt each other

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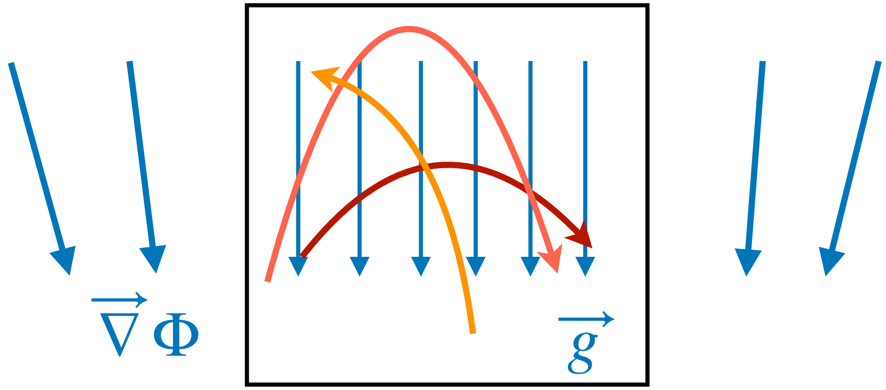

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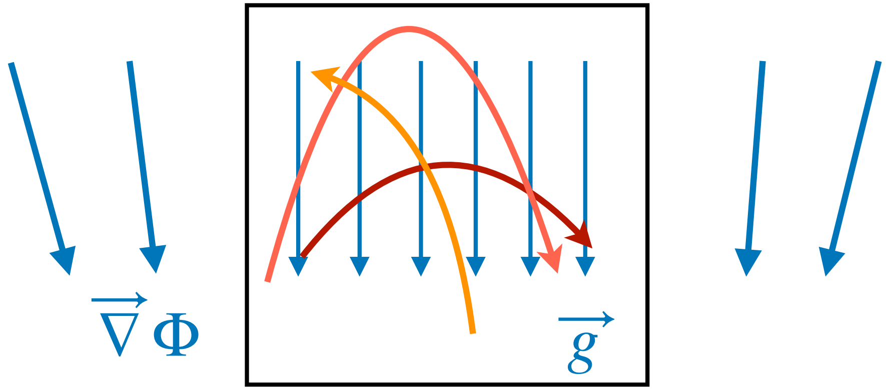
Effect of gravity in the box is identical to the effect of inertial forces in a frame accelerating with $\vec{a} = -\vec{g}$

Equivalence principle

WEP 3

The motions of free-falling masses are the same in an uniformly accelerated frame and in a gravitational field in a sufficiently small region of the spacetime



It is always possible to get rid of a gravitational field **locally** by passing to a free-falling frame

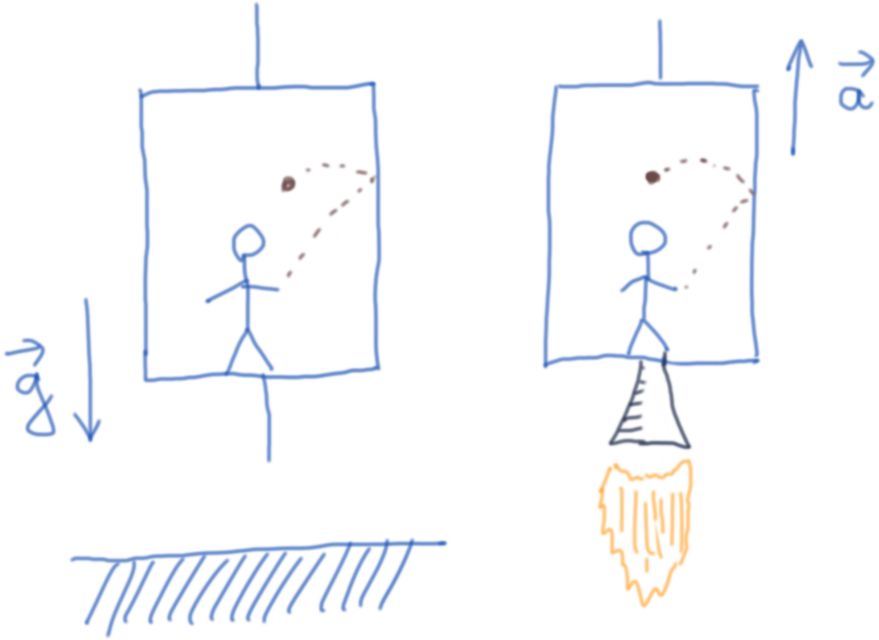
$$-\frac{\partial \Phi}{\partial x^i} = \vec{g}_0 - \frac{\partial^2 \Phi}{\partial x^i \partial x^j}(x^j - x_0^j) + O(\Delta x^2)$$

const

tidal forces

Equivalence principle

Einstein's elevator



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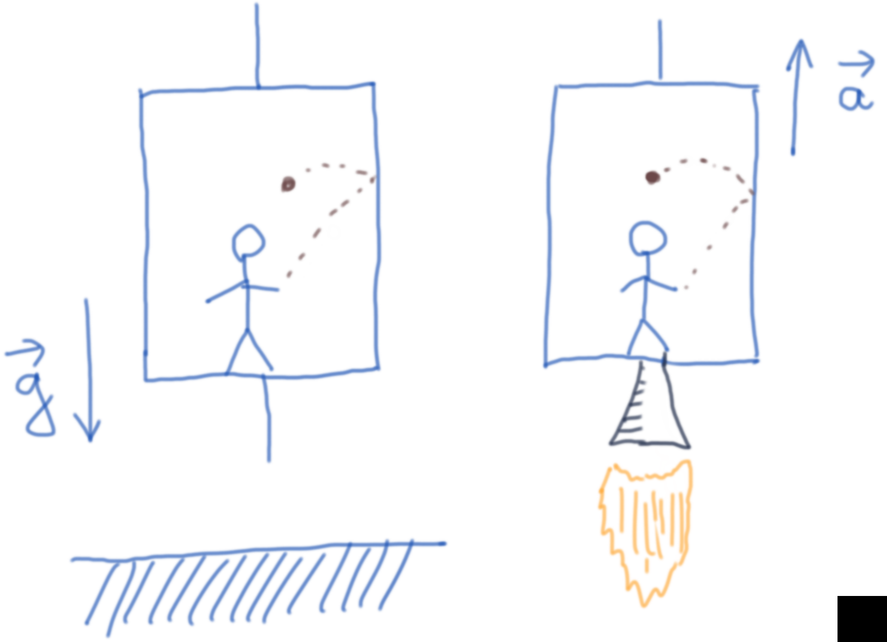
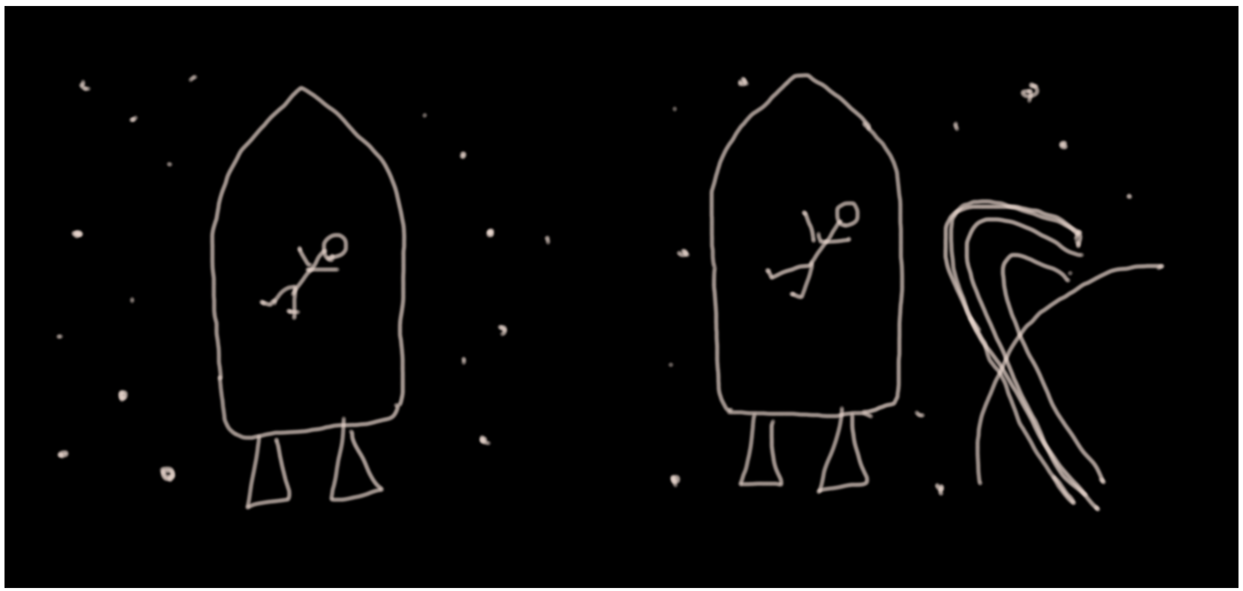


Rocket *gedankenexperiment*

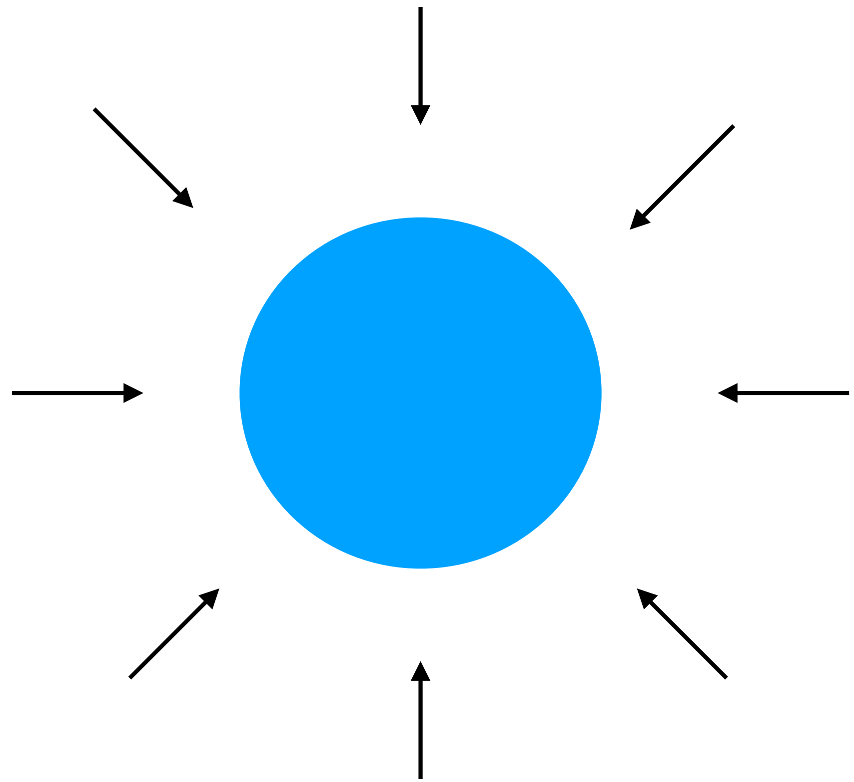


Equivalence principle

In order to reconcile gravity and SR we need the counterpart of inertial frames

What are really the inertial frames if gravitational forces are present?

Not the frame related to the Earth's crust…



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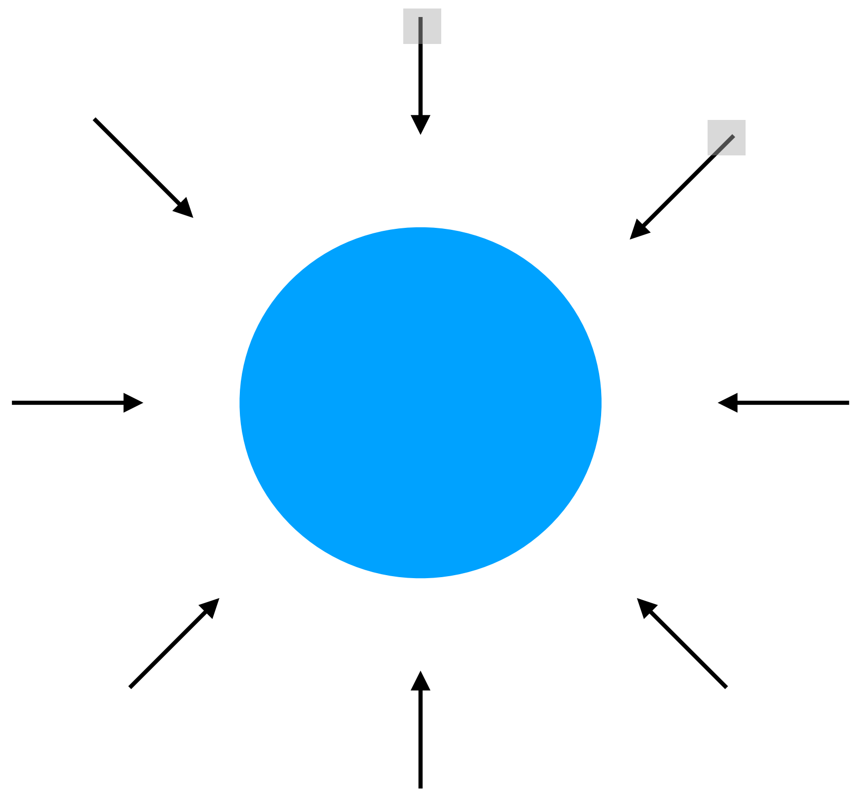

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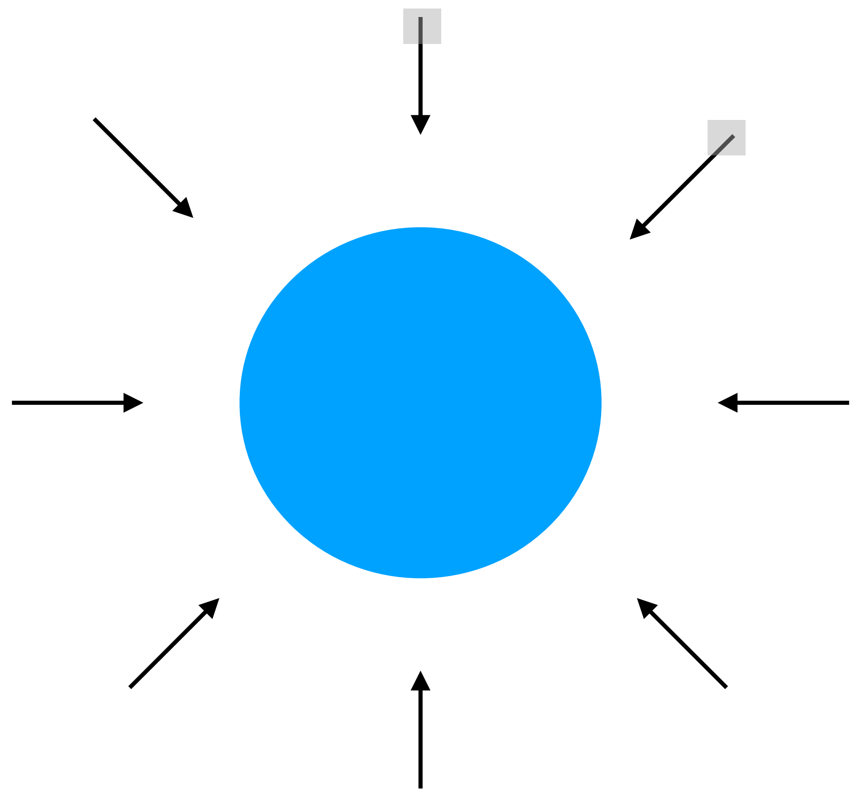

Einstein's equivalence principle

In small regions of spacetime, in local inertial frames, all laws of physics reduce to their special relativity form (electromagnetism, thermodynamics etc.)

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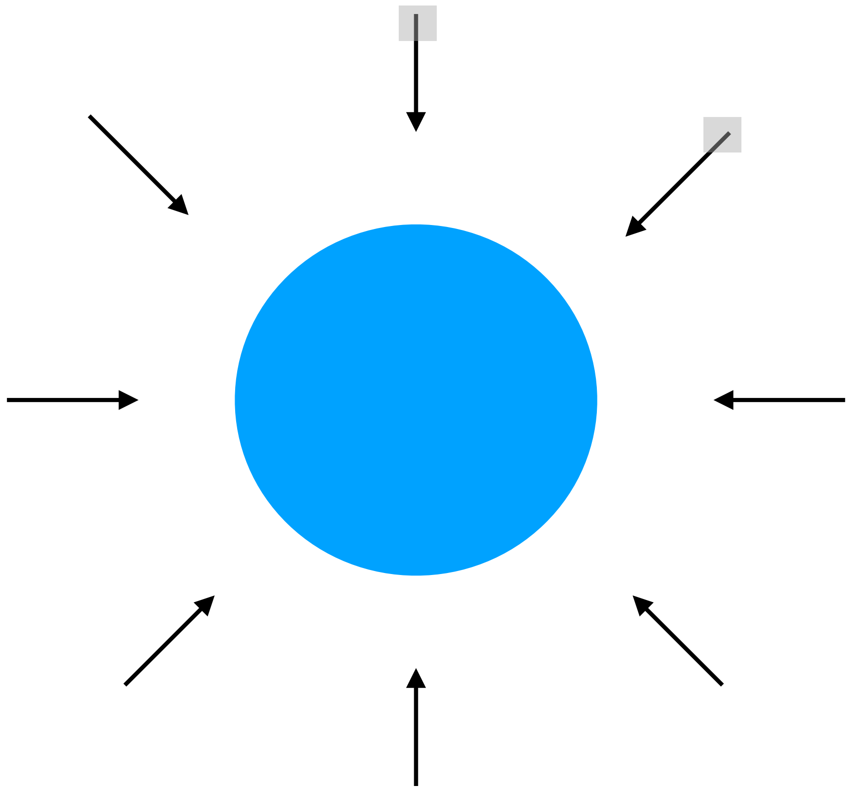

Einstein's equivalence principle

In small regions of spacetime, in local inertial frames, all laws of physics reduce to their special relativity form (electromagnetism, thermodynamics etc.)

Gravity is not a force, but rather an effect of free-falling (inertial frames) at various points being incompatible with each other

Mach's principle

special relativity, Newtonian mechanics

Inertial frames, inertial forces, non-rotating frames = fixed,
determined by the structure of the spacetime

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Ernst Mach (1838-1916)

Matter (motions and distribution of masses in the Universe) should contribute to the local definitions of inertial frame, accelerating frame, non-rotating frame…

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A. Einstein

Motions and the distribution of matter determines the structure/geometry of the spacetime, and thus also the notion of inertial frames

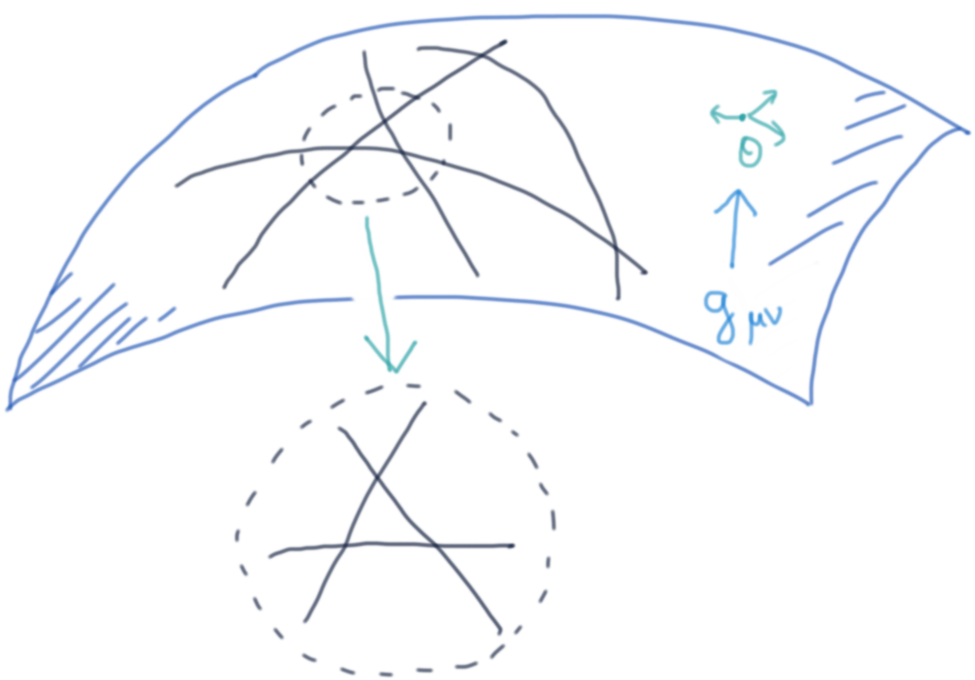
GR is consistent with some, but not all Mach's intuitions.

Non-euclidean (curved) geometries

Euclidean geometry: based on the notion of points and straight lines

Gauss, Bolyai, Lobachevskii - non-Euclidean geometry (early 1800)

Riemann 1850's-60's - general theory of non-Euclidean spaces of any dimension and curvature



point-dependent metric $g_{\mu\nu}(x)$, defines angles and distances

set of special curves through each point and in any direction (geodesics)

…but small vicinity of any point looks like a flat geometry!

Non-euclidean (curved) geometries

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Spacetime with gravity

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geometry looks locally flat in an (infinitesimally small) neighbourhood of any point	physics looks consistent with SR in an (infinitesimal small) neighbourhood of any point (local inertial frame)

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Idea: gravity is the effect of the spacetime geometry being curved