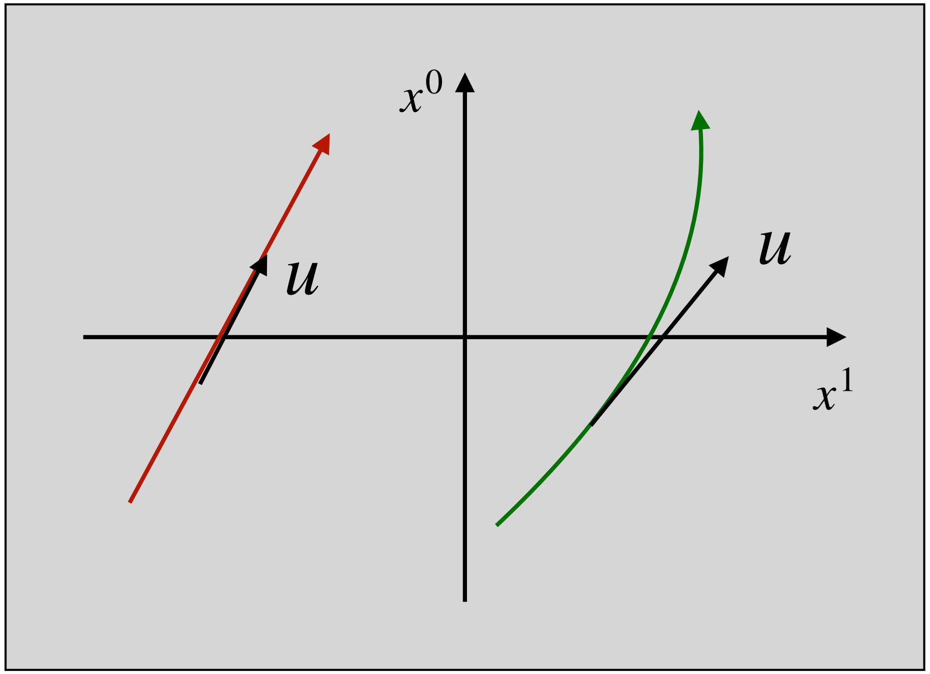
Kinematics in SR



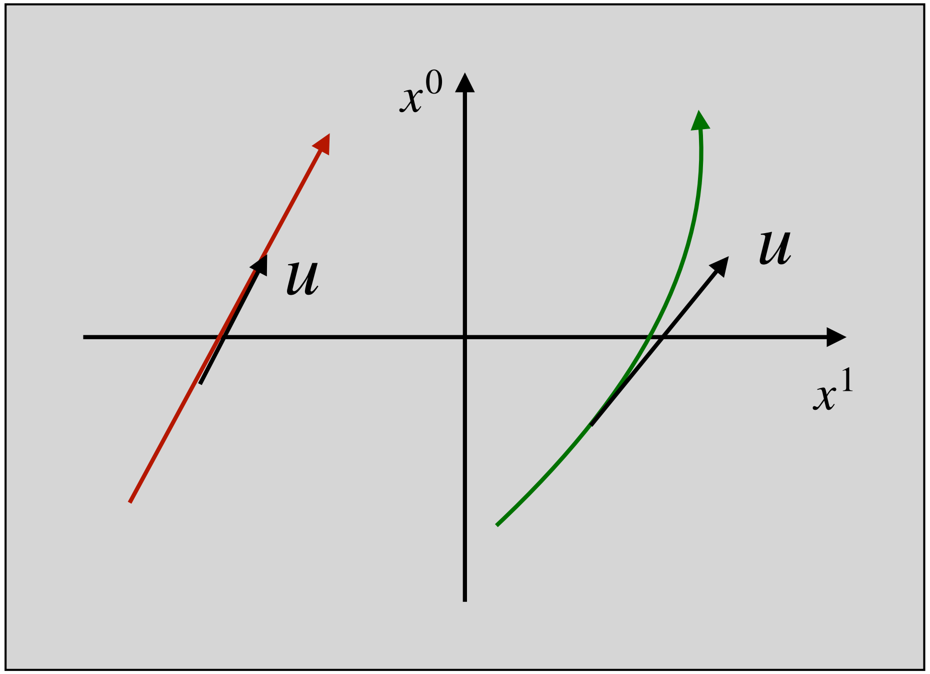
4-velocity u^μ

normalized $u \cdot u = -1$

future-pointing $u^0 > 0$

any co-moving frame has $e_0 = u$

Kinematics in SR



4-velocity u^μ

normalized $u \cdot u = -1$

future-pointing $u^0 > 0$

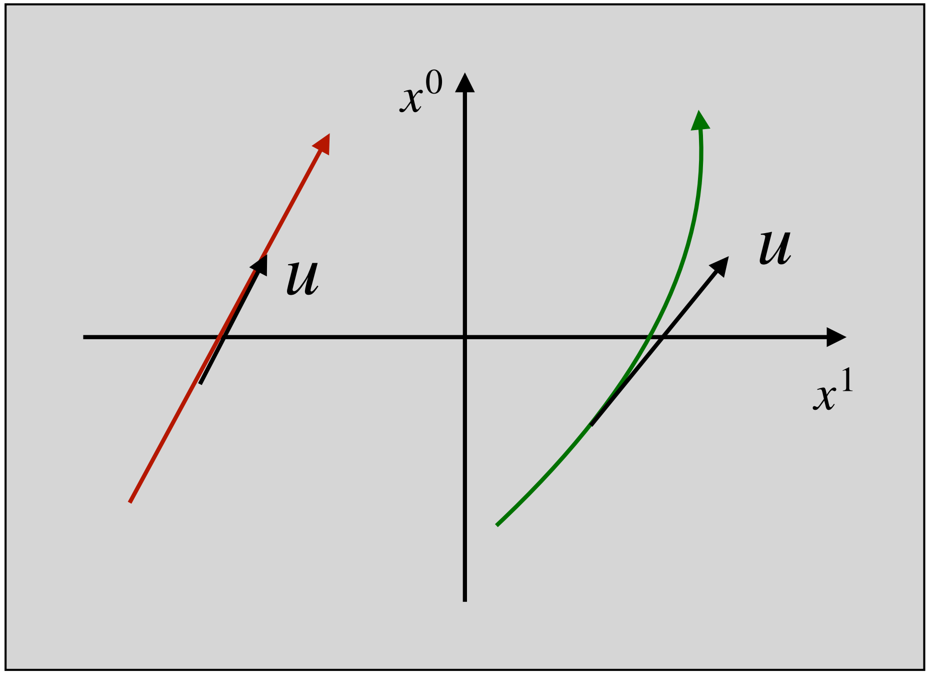
any co-moving frame has $e_0 = u$

proper time τ:

its flow matches the coordinate time of the co-moving frame

interpretation: time measured by a moving perfect clock

Kinematics in SR



$$d\tau = d\tilde{x}^0 \qquad \tilde{e}_0^{\mu} = u^{\mu} = \begin{pmatrix} \gamma \\ \gamma v^i \end{pmatrix}$$

$$x^0 = \gamma \tilde{x}^0 + \gamma \vec{v} \cdot \vec{\tilde{x}} \implies dx^0 = \gamma \, d\tilde{x}^0 = \gamma \, d\tau$$

4-velocity u^{μ}

normalized $u \cdot u = -1$

future-pointing $u^0 > 0$

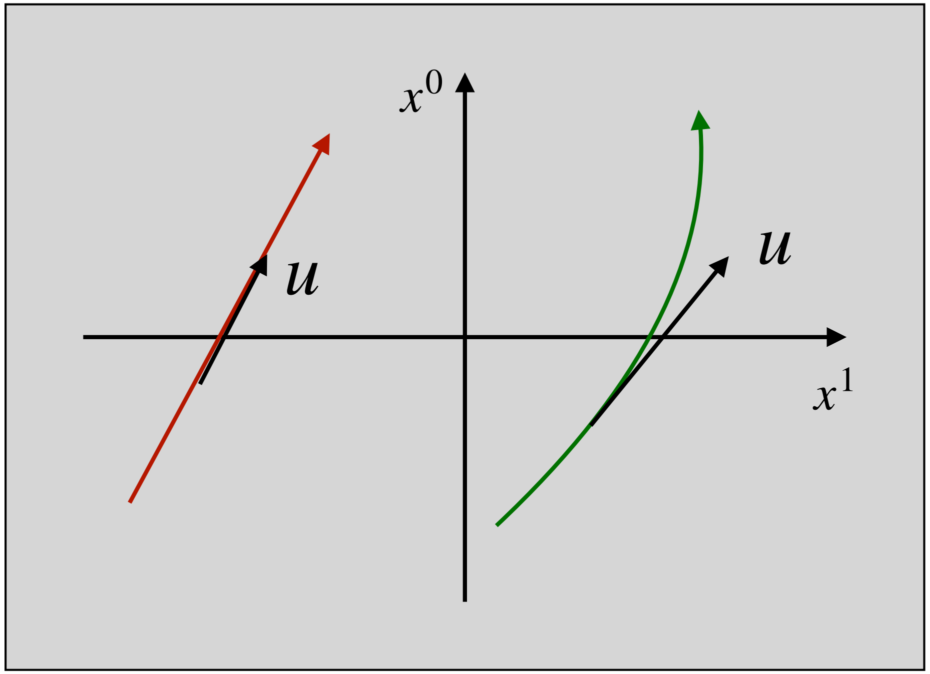
any co-moving frame has $e_0 = u$

proper time τ:

its flow matches the coordinate time of the co-moving frame

interpretation: time measured by a moving perfect clock

Kinematics in SR



$$d\tau = d\tilde{x}^0 \qquad \tilde{e}_0^\mu = u^\mu = \begin{pmatrix} \gamma \\ \gamma v^i \end{pmatrix}$$

$$x^0 = \gamma \tilde{x}^0 + \gamma \vec{v} \cdot \vec{\tilde{x}} \implies dx^0 = \gamma \, d\tilde{x}^0 = \gamma \, d\tau$$

$$d\tau = \gamma^{-1} \, dx^0 \qquad \tau(x^0) = \tau_0 + \int_{t_0}^{x^0} \sqrt{1 - v^2} \, dx^0$$

4-velocity u^μ

normalized $u \cdot u = -1$

future-pointing $u^0 > 0$

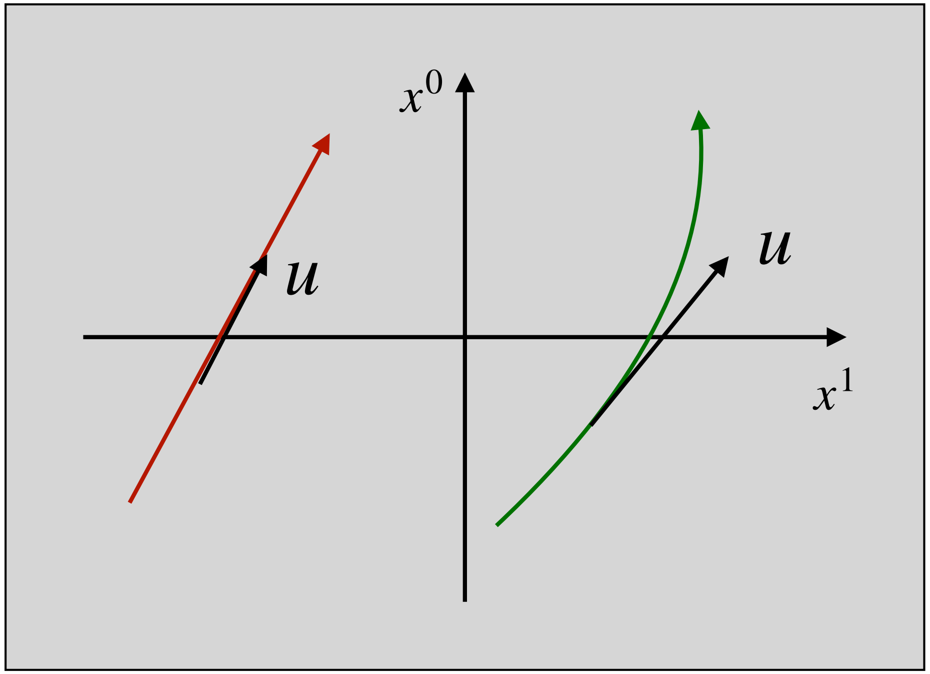
any co-moving frame has $e_0 = u$

proper time τ:

its flow matches the coordinate time of the co-moving frame

interpretation: time measured by a moving perfect clock

Kinematics in SR



$$d\tau = d\tilde{x}^0 \qquad \tilde{e}_0^\mu = u^\mu = \begin{pmatrix} \gamma \\ \gamma v^i \end{pmatrix}$$

$$x^0 = \gamma \tilde{x}^0 + \gamma \vec{v} \cdot \vec{\tilde{x}} \implies dx^0 = \gamma\, d\tilde{x}^0 = \gamma\, d\tau$$

$$d\tau = \gamma^{-1}\, dx^0 \qquad \tau(x^0) = \tau_0 + \int_{t_0}^{x^0} \sqrt{1 - v^2}\, dx^0$$

4-velocity u^μ

normalized $\quad u \cdot u = -1$

future-pointing $\quad u^0 > 0$

any co-moving frame has $e_0 = u$

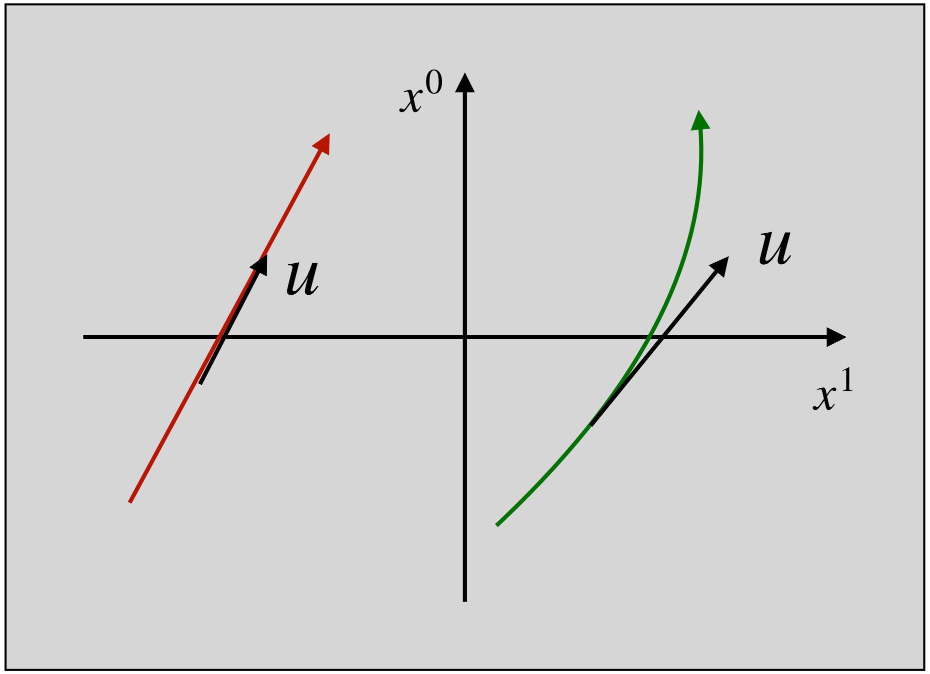
proper time τ:

its flow matches the coordinate time of the co-moving frame

interpretation: time measured by a moving perfect clock

defined intrinsically, by the motion of the body

Kinematics in SR



$$d\tau = d\tilde{x}^0 \qquad \tilde{e}^\mu_0 = u^\mu = \begin{pmatrix} \gamma \\ \gamma v^i \end{pmatrix}$$

$$x^0 = \gamma \tilde{x}^0 + \gamma \vec{v} \cdot \vec{\tilde{x}} \implies dx^0 = \gamma\, d\tilde{x}^0 = \gamma\, d\tau$$

$$d\tau = \gamma^{-1}\, dx^0 \qquad \tau(x^0) = \tau_0 + \int_{t_0}^{x^0} \sqrt{1 - v^2}\, dx^0$$

$$x^\mu(\tau) \qquad \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dx^0}\frac{dx^0}{d\tau} = \begin{pmatrix} 1 \\ v^i \end{pmatrix} \gamma = u^\mu$$

4-velocity u^μ

normalized $u \cdot u = -1$

future-pointing $u^0 > 0$

any co-moving frame has $e_0 = u$

proper time τ:

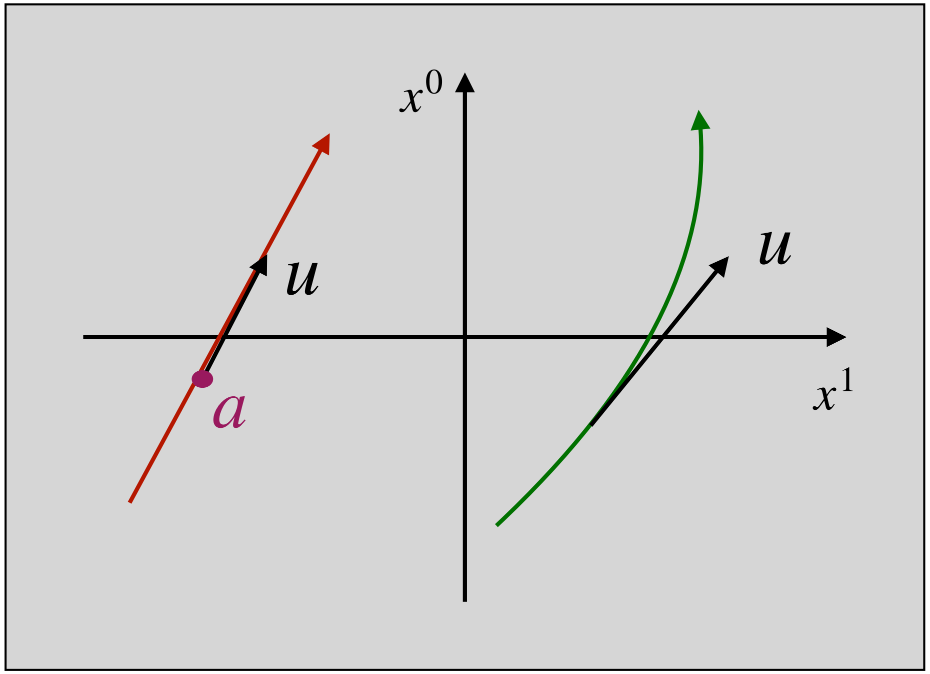
its flow matches the coordinate time of the co-moving frame

interpretation: time measured by a moving perfect clock

defined intrinsically, by the motion of the body

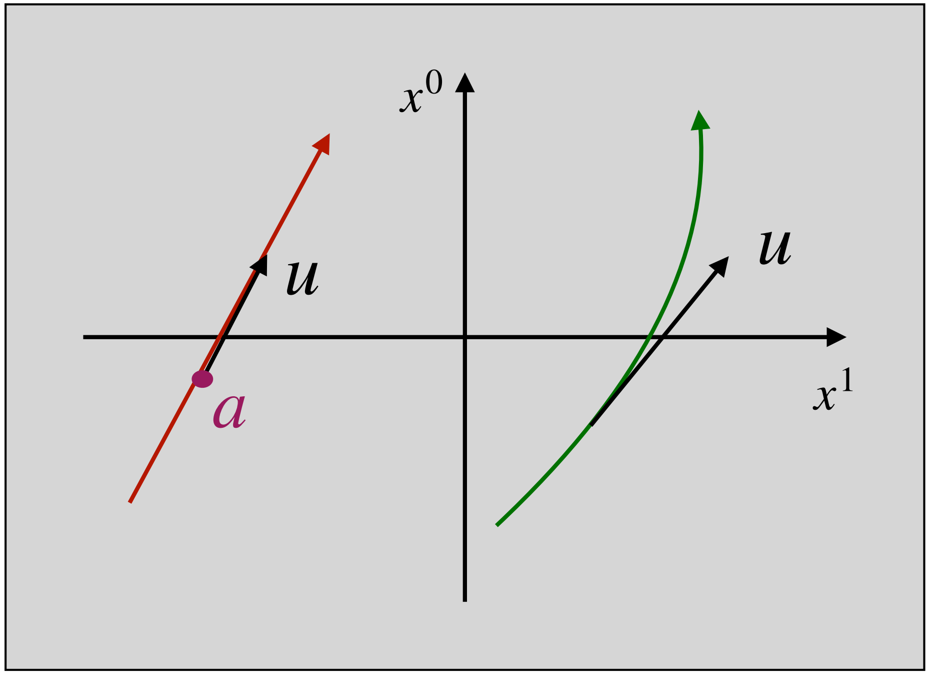
natural parametrization related to 4-velocity

Kinematics in SR



massive particle moving with constant velocity

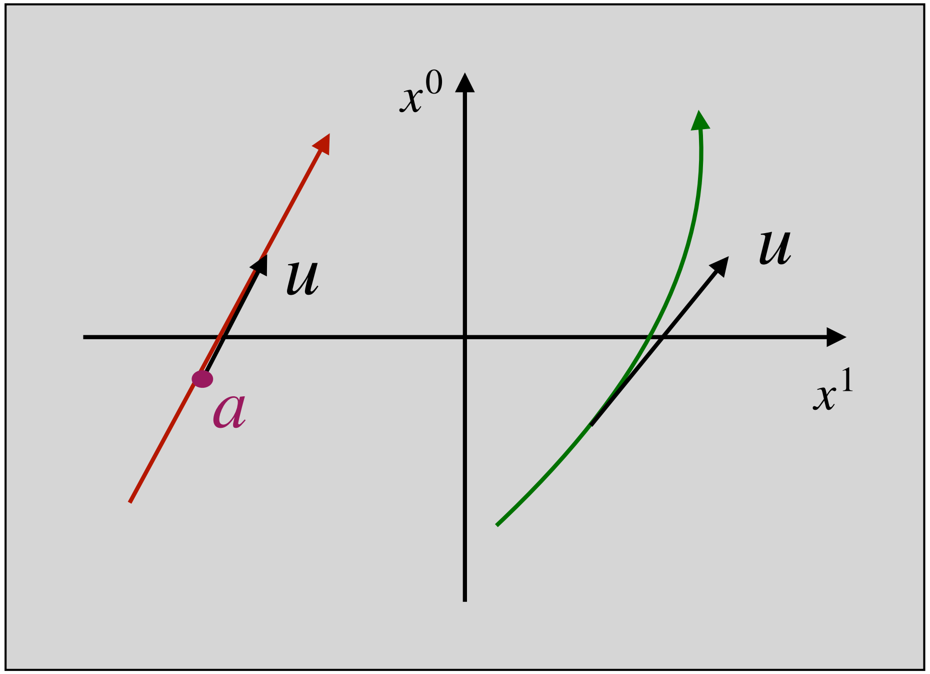
Kinematics in SR



massive particle moving with constant velocity

$$x^\mu(\tau) = a^\mu + \tau\, u^\mu$$

Kinematics in SR



massive particle moving with constant velocity

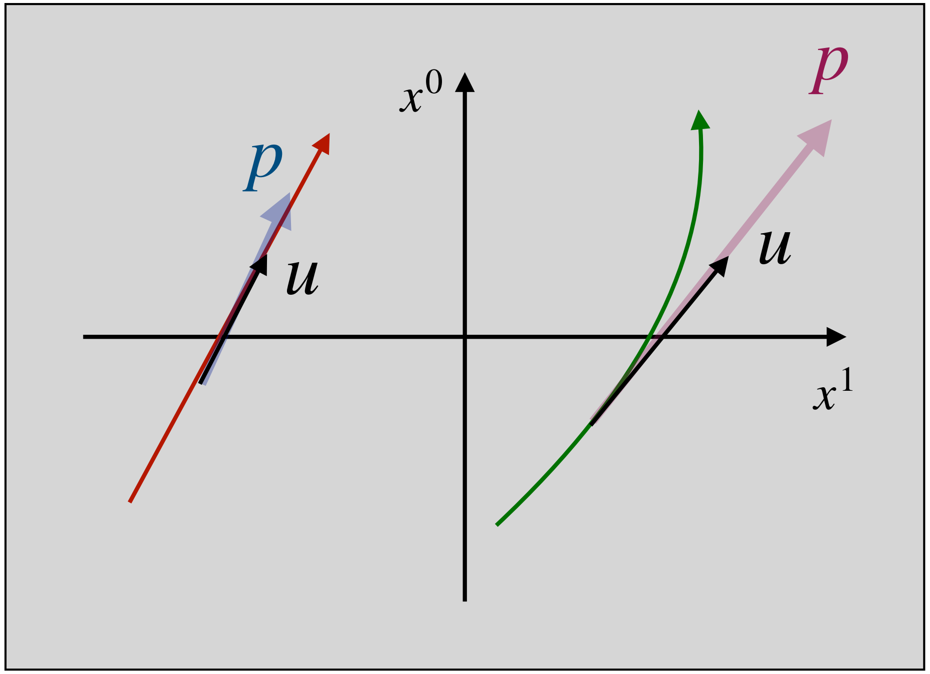
$$x^\mu(\tau) = a^\mu + \tau\, u^\mu$$

reparametrization:

$$\tau \to \tilde{\tau} = \tau + D$$

$$a \to \tilde{a} = a - D\,u$$

Kinematics in SR



4-momentum

particle of rest mass m

$$p^\mu := m\, u^\mu = \begin{pmatrix} \gamma m \\ \gamma m v^i \end{pmatrix}$$

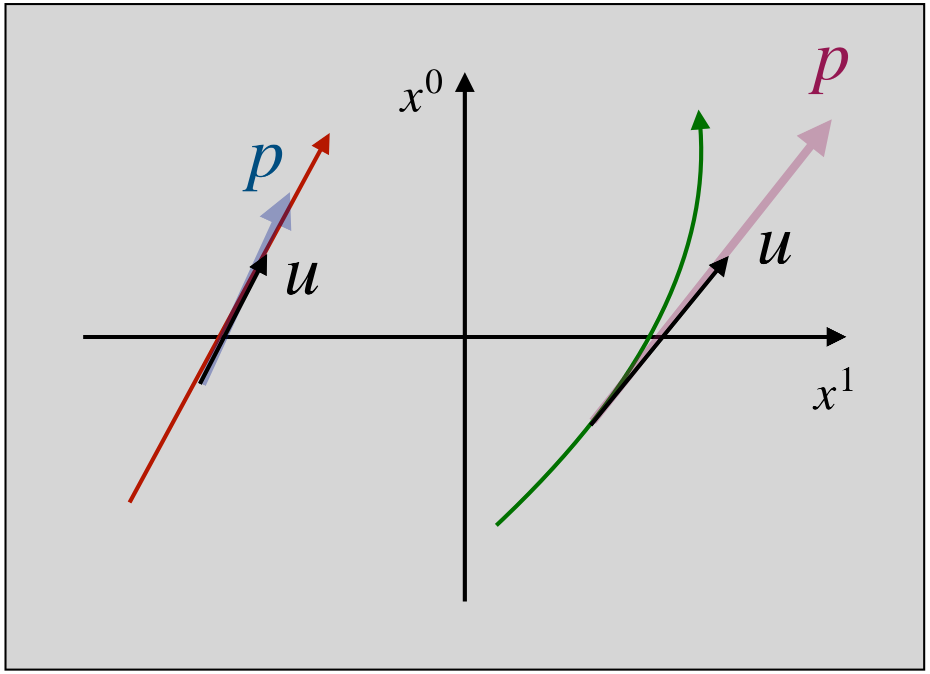
energy

momentum

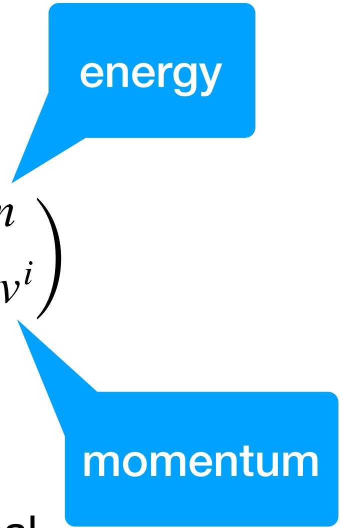
$$p \cdot p = -\, m^2$$

total 4-momentum conserved by local forces

Kinematics in SR



4-momentum



particle of rest mass m

$$p^\mu := m\, u^\mu = \begin{pmatrix} \gamma m \\ \gamma m v^i \end{pmatrix}$$

$$p \cdot p = -m^2$$

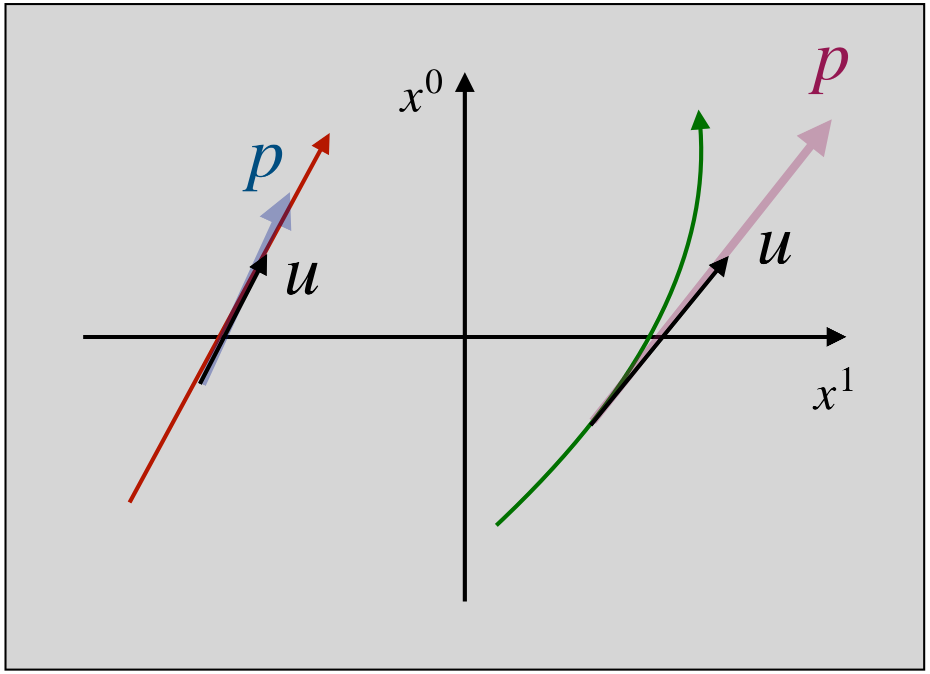
total 4-momentum conserved by local forces

small velocity limit $v \ll 1$ $\qquad v_{old}/c \ll 1$

$$\gamma = (1 - \vec{v}^2)^{-1/2} = 1 + \frac{1}{2}\vec{v}^2 + O(v^3)$$

$$p^\mu = \begin{pmatrix} m + \frac{1}{2} m \vec{v}^2 + O(v^3) \\ m v^i + O(v^3) \end{pmatrix}$$

Kinematics in SR



4-momentum

particle of rest mass m

$$p^\mu := m\,u^\mu = \begin{pmatrix} \gamma m \\ \gamma m v^i \end{pmatrix}$$

energy

momentum

$$p \cdot p = -m^2$$

total 4-momentum conserved by local forces

small velocity limit $v \ll 1$ $\qquad v_{old}/c \ll 1$

$$\gamma = (1 - \vec{v}^2)^{-1/2} = 1 + \frac{1}{2}\vec{v}^2 + O(v^3)$$

$$p^\mu = \begin{pmatrix} m + \frac{1}{2} m \vec{v}^2 + O(v^3) \\ m v^i + O(v^3) \end{pmatrix}$$

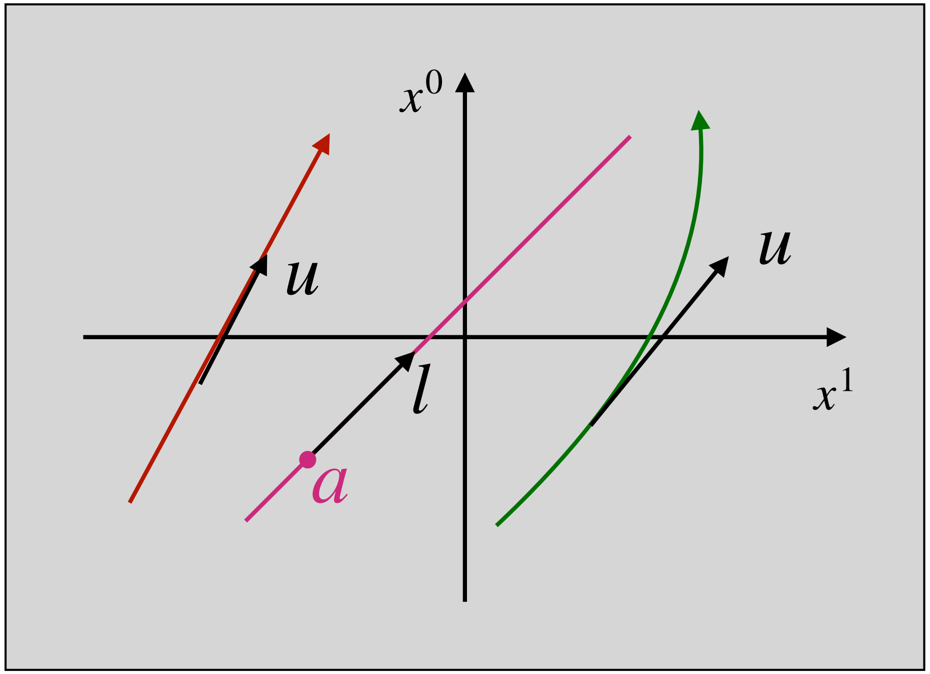
Re-introducing c

$$p^\mu_{new} = \begin{pmatrix} m + \frac{1}{2c^2} m \vec{v}^2_{old} + O\left((v_{old}/c)^3\right) \\ m v^i_{old}/c + O\left((v_{old}/c)^3\right) \end{pmatrix}$$

$$E_{old} = c^2 p^0 = mc^2 + \frac{1}{2} m \vec{v}^2_{old} + O\left((v_{old}/c)^3\right)$$

$$p^i_{old} = c\,p^i = m v^i_{old} + O\left((v_{old}/c)^3\right)$$

Kinematics in SR

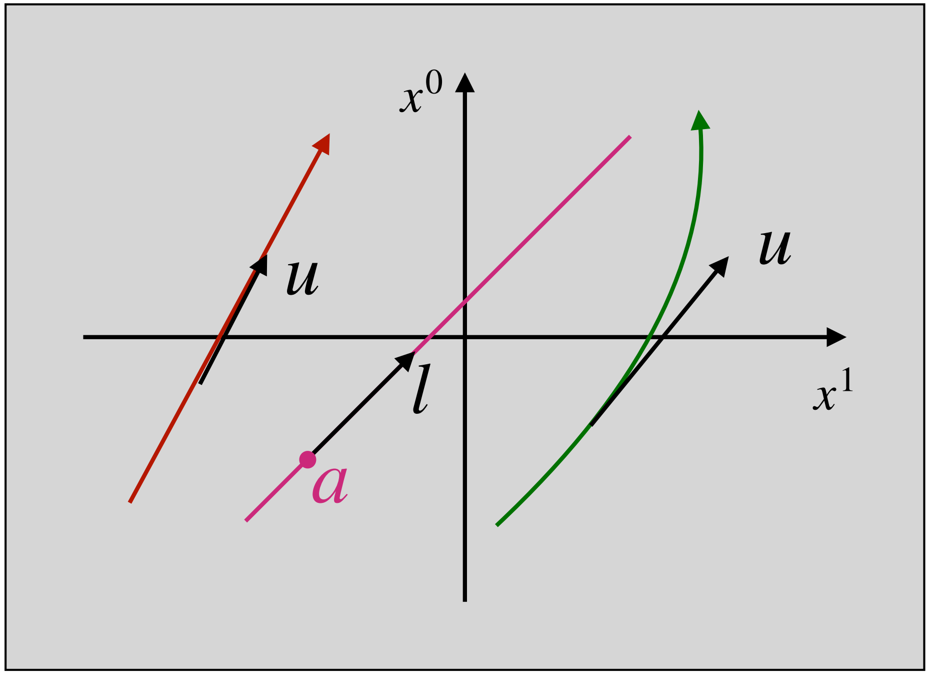


light rays (worldlines of photons)

$$x^\mu(\lambda) = a^\mu + \lambda\, l^\mu \qquad l \cdot l = 0$$

$$-\left(l^0\right)^2 + \sum_i l^i\, l^i = 0$$

Kinematics in SR



light rays (worldlines of photons)

$$x^\mu(\lambda) = a^\mu + \lambda\, l^\mu \qquad l \cdot l = 0$$

$$-\left(l^0\right)^2 + \sum_i l^i\, l^i = 0$$

velocity = speed of light, i.e. 1

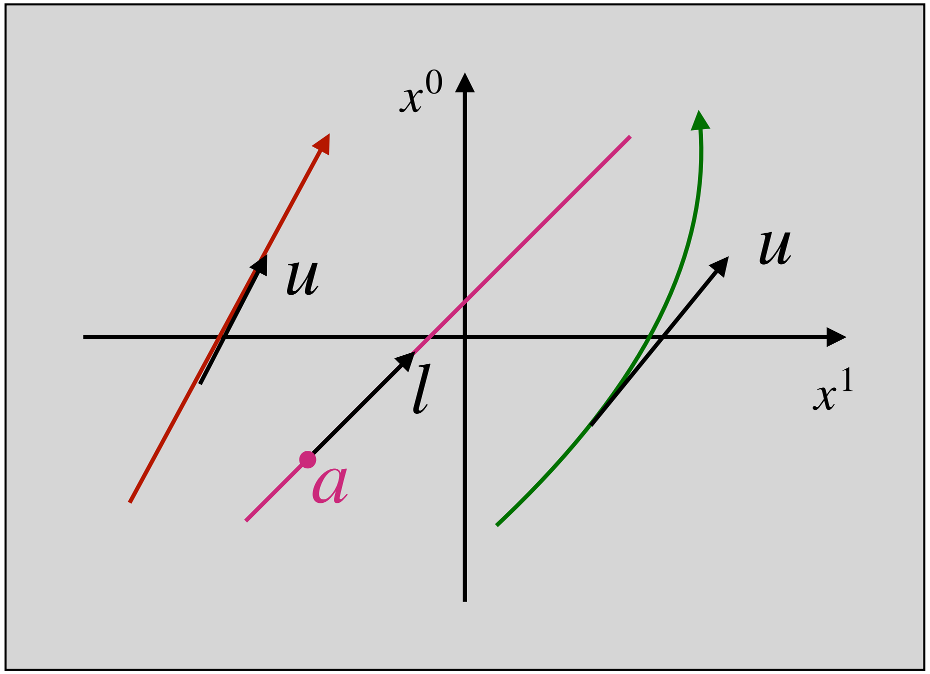
$$\frac{dx^i}{dx^0} = \frac{dx^i}{d\lambda} \cdot \frac{d\lambda}{dx^0} = \frac{l^i}{l^0}$$

$$\sum_i \frac{l^i}{l^0}\,\frac{l^i}{l^0} = 1$$

$$l^0 = \sqrt{\sum_i l^i\, l^i}$$

…in any inertial frame!

Kinematics in SR



light rays (worldlines of photons)

$$x^\mu(\lambda) = a^\mu + \lambda\, l^\mu \qquad\qquad l \cdot l = 0$$

$$-\left(l^0\right)^2 + \sum_i l^i\, l^i = 0$$

velocity = speed of light, i.e. 1

$$\frac{dx^i}{dx^0} = \frac{dx^i}{d\lambda} \cdot \frac{d\lambda}{dx^0} = \frac{l^i}{l^0}$$

$$\sum_i \frac{l^i}{l^0}\,\frac{l^i}{l^0} = 1$$

$$l^0 = \sqrt{\sum_i l^i\, l^i}$$

…in any inertial frame!

affine reparametrizations

$$\lambda \to \tilde{\lambda} = C\lambda + D$$

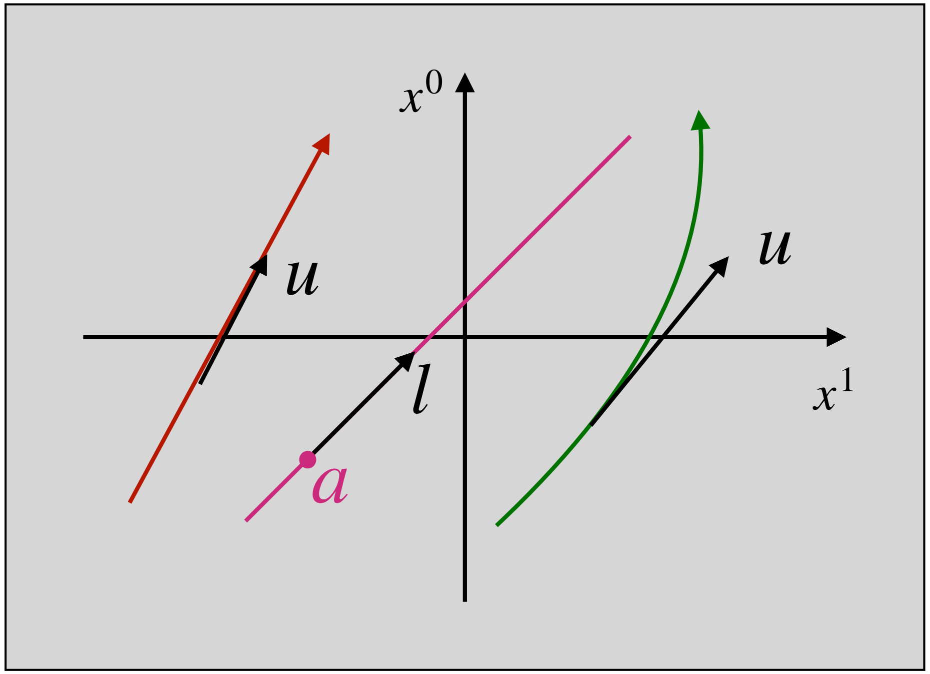
$$a \to \tilde{a} = a - D\, l$$

$$l \to \tilde{l} = \frac{1}{C}\, l$$

l not normalizable

$\Longrightarrow$ no 4-velocity or proper time for photons

Kinematics in SR



affine reparametrizations

$$\lambda \to \tilde{\lambda} = C\lambda + D$$

$$a \to \tilde{a} = a - D\,l$$

$$l \to \tilde{l} = \frac{1}{C}\,l$$

l not normalizable

$\Longrightarrow$ no 4-velocity or proper time for photons

light rays (worldlines of photons)

$$x^\mu(\lambda) = a^\mu + \lambda\, l^\mu \qquad l \cdot l = 0$$

$$-\left(l^0\right)^2 + \sum_i l^i\, l^i = 0$$

velocity = speed of light, i.e. 1

$$\frac{dx^i}{dx^0} = \frac{dx^i}{d\lambda} \cdot \frac{d\lambda}{dx^0} = \frac{l^i}{l^0}$$

$$\sum_i \frac{l^i}{l^0}\,\frac{l^i}{l^0} = 1$$

$$l^0 = \sqrt{\sum_i l^i\, l^i}$$

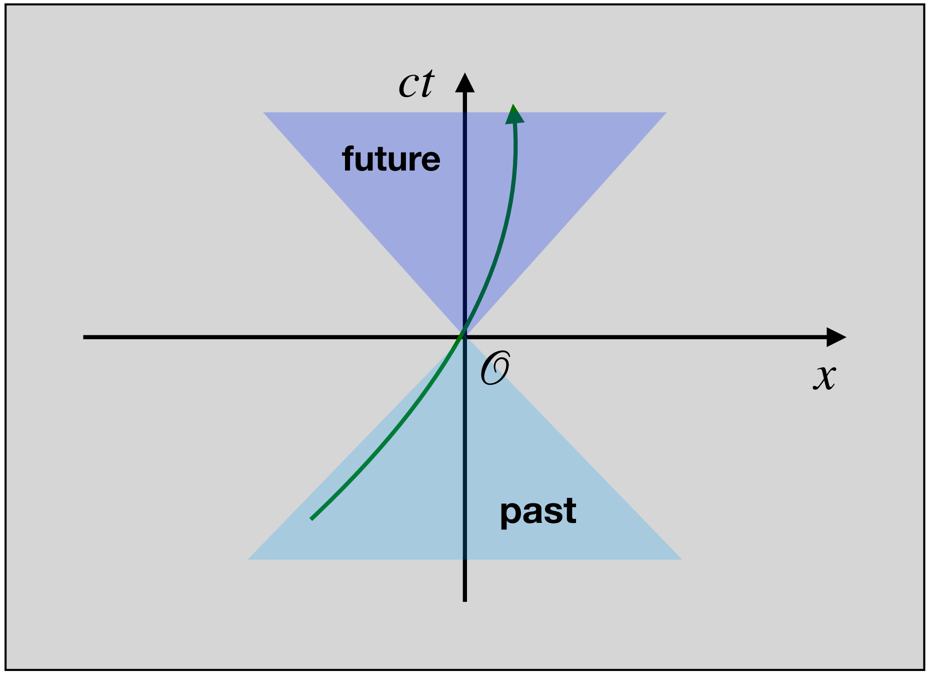
…in any inertial frame!

photon's 4-momentum

$$p^\mu := \frac{E}{l^0}\, l^\mu \qquad p \cdot p = 0$$

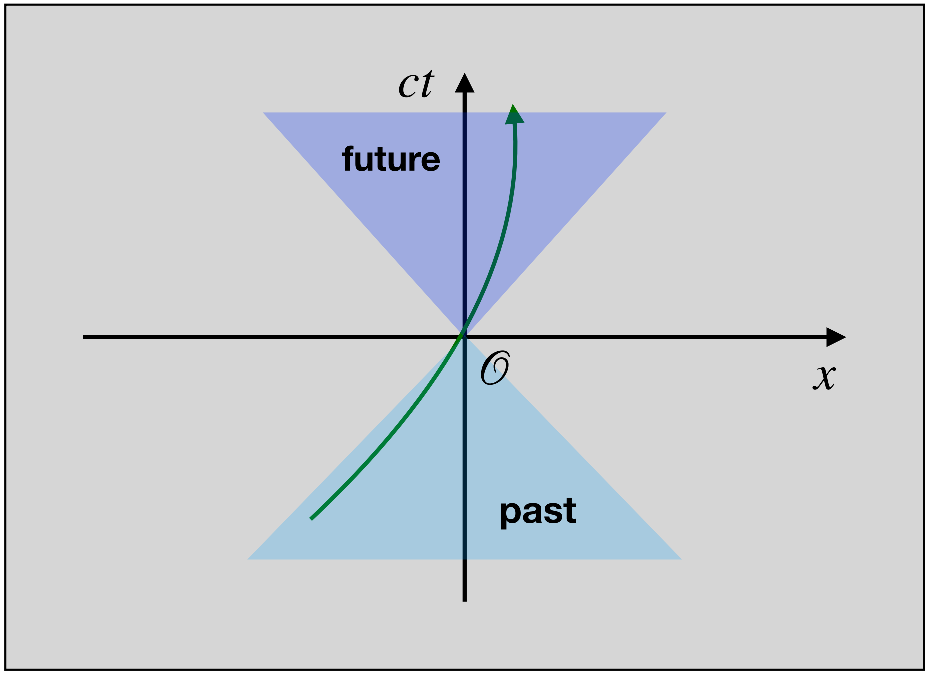
$$p^\mu = \begin{pmatrix} E \\ p^i \end{pmatrix} \qquad E = |\,\vec{p}\,|$$

Kinematics in SR



light cones, causality

Kinematics in SR



light cones, causality

$$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} < 0$$ **timelike**

Inside the light cone

Worldlines of massive particles

Future/past distinction

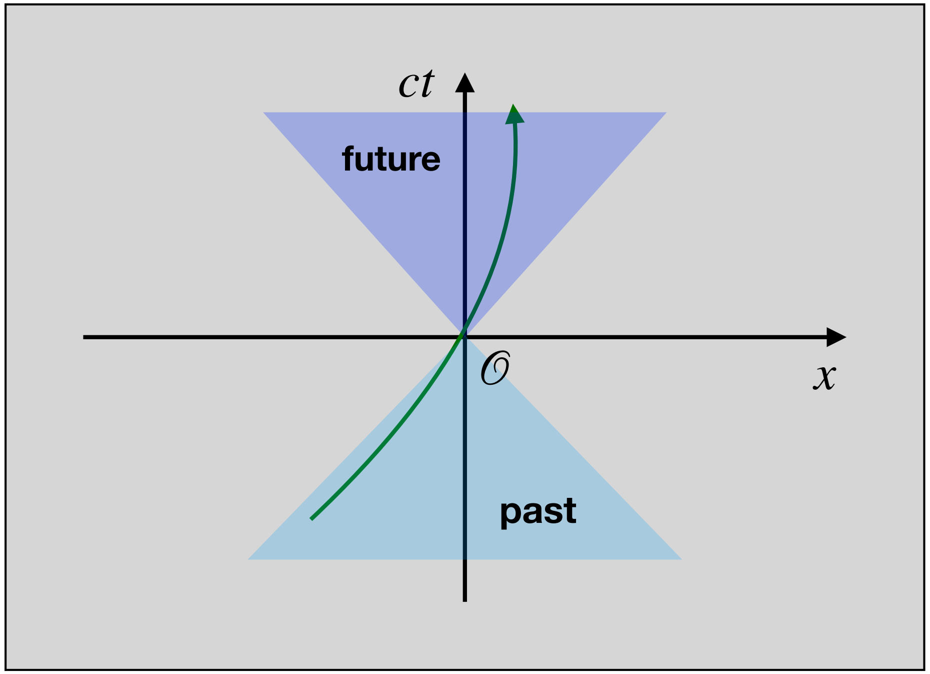
$$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} = 0$$ **null**

The light cone

Points connected by light rays

Future/past distinction

Kinematics in SR



$$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} > 0$$ **spacelike**

Outside the light cone

No future/past distinction (somewhere else)

light cones, causality

$$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} < 0$$ **timelike**

Inside the light cone

Worldlines of massive particles

Future/past distinction

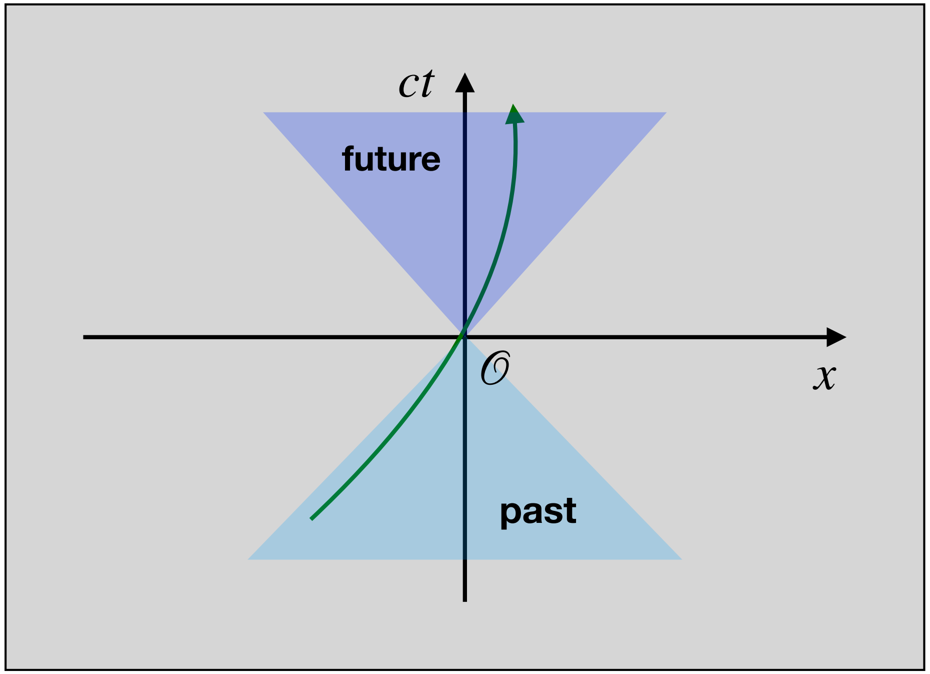
$$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} = 0$$ **null**

The light cone

Points connected by light rays

Future/past distinction

Kinematics in SR



light cones, causality

$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} < 0$ **timelike**

Inside the light cone

Worldlines of massive particles

Future/past distinction

$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} = 0$ **null**

The light cone

Points connected by light rays

Future/past distinction

$\Delta x^\mu \, \Delta x^\nu \, \eta_{\mu\nu} > 0$ **spacelike**

Outside the light cone

No future/past distinction (somewhere else)

No superluminal interactions: no worldlines outside the light cone

events outside past light cone cannot influence $\mathcal{O}$

$\mathcal{O}$ cannot influence events outside future light cone

End of lecture 1