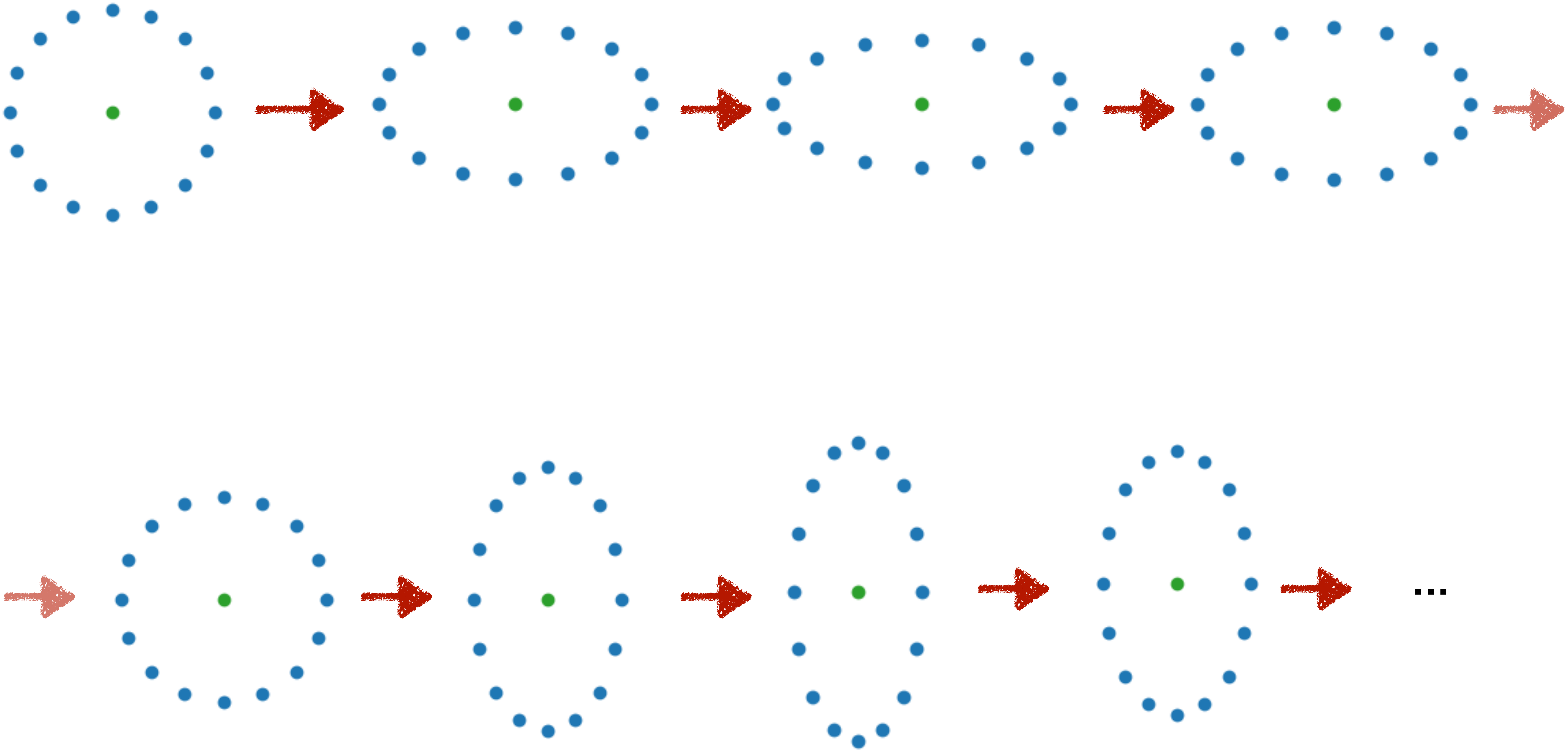
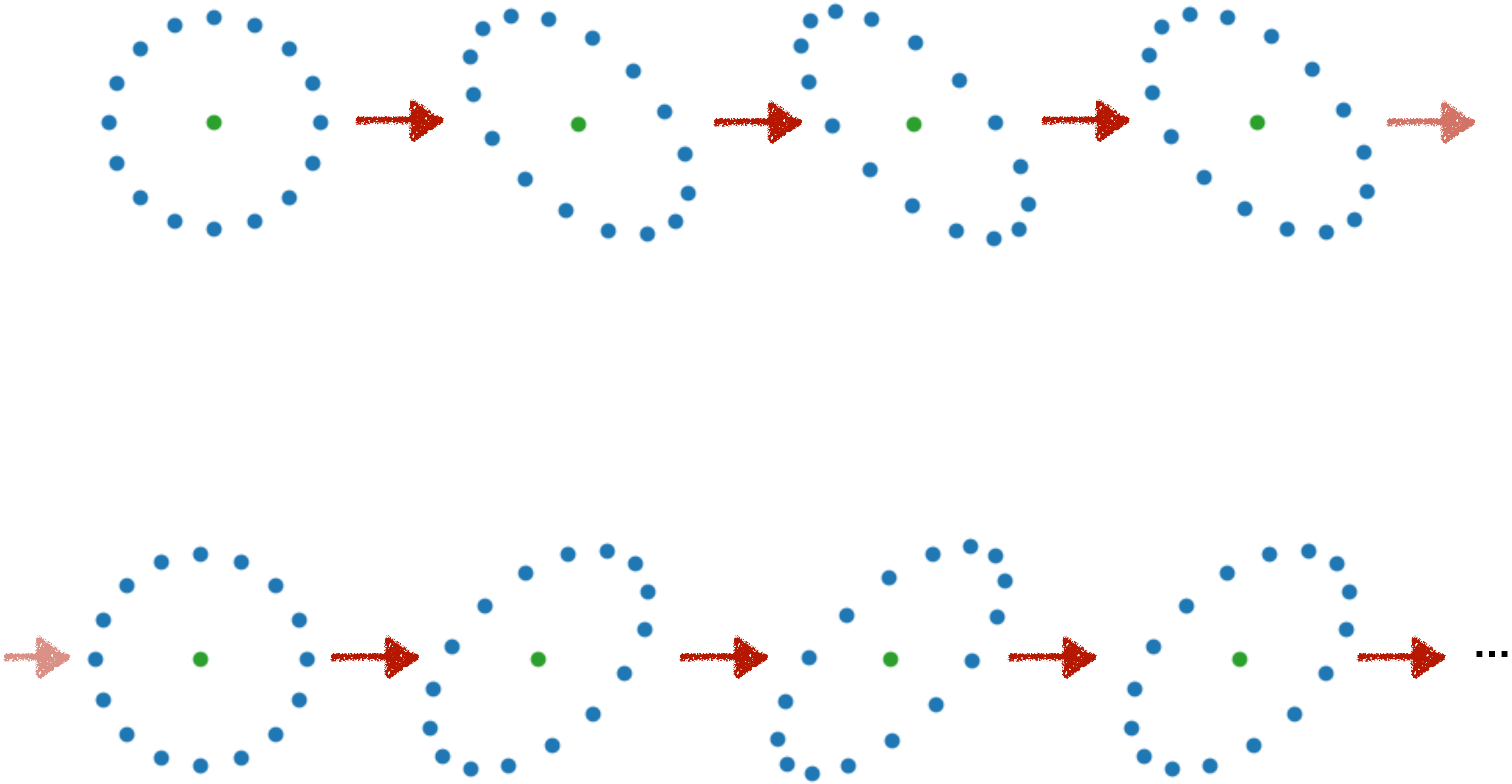
Gravitational waves

+ mode



Gravitational waves

X mode



Gravitational waves

Energy flux of a plane wave

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Fundamental problem with a local energy density of gravitational field

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…but for GW at perturbative level there's a reasonable expression

$$F = \frac{h_0^2\, \omega^2}{32\pi G} = \frac{\pi f^2}{8G}$$

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B. Schutz *„A First Course in General Relativity"* Ch. 9.4, p. 235

interaction of a GW with harmonic oscillators

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effective stress-energy tensor for gravitational waves $t_{\mu\nu} = \dfrac{1}{32\pi G} \left\langle \partial_\mu h_{\alpha\beta}^{TT}\, \partial_\nu h_{TT}^{\alpha\beta} \right\rangle$

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1. Vacuum Einstein equations up to 2nd order

$$g_{\mu\nu} = \eta_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu}$$

$$R_{\mu\nu} = \underbrace{R^{(1)}_{\mu\nu}}_{\text{linearized Ricci}} + \underbrace{R^{(2)}_{\mu\nu}}_{\text{quadratic term in the expansion of Ricci}}$$

$$R^{(1)}_{\mu\nu}\left[h^{(1)}_{\mu\nu}\right] = 0$$

$$R^{(1)}_{\mu\nu}\left[h^{(2)}_{\mu\nu}\right] + R^{(2)}\left[h^{(1)}_{\mu\nu}\right] = 0$$

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2. Re-writing the equations for $h^{(2)}_{\mu\nu}$

$$R^{(1)}_{\mu\nu}\left[h^{(2)}_{\alpha\beta}\right] - \frac{1}{2}\eta_{\mu\nu}\,\eta^{\kappa\lambda}R^{(1)}_{\kappa\lambda}\left[h^{(2)}_{\alpha\beta}\right] = 8\pi G\, t^{\text{loc}}_{\mu\nu}$$

Gravity gravitates!

$$t_{\mu\nu} = -\frac{1}{8\pi G}\left(R^{(2)}_{\mu\nu}\left[h^{(1)}_{\alpha\beta}\right] - \frac{1}{2}\eta_{\mu\nu}\,\eta^{\kappa\lambda}R^{(2)}_{\kappa\lambda}\left[h^{(1)}_{\alpha\beta}\right]\right)$$

Higher order terms in metic „feel” the gravity of a GW

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This $t_{\mu\nu}^{\text{loc}}$ is

- local
- conserved: $\partial_\mu t^{\mu\nu}_{\text{loc}} = 0$
- gauge-dependent

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3. Average over many wavelengths

$$\langle \dots \rangle = V_D^{-1} \int_D (\dots)\, d^4x$$

$$t_{\mu\nu} = \langle t_{\mu\nu}^{\text{loc}} \rangle$$

$$t_{\mu\nu} = \frac{1}{32\pi G} \left\langle \partial_\mu h_{\alpha\beta}^{TT}\, \partial_\nu h_{TT}^{\alpha\beta} \right\rangle$$

$$t^{0z} = F = \frac{h_0^2\, \omega^2}{32\pi G}$$