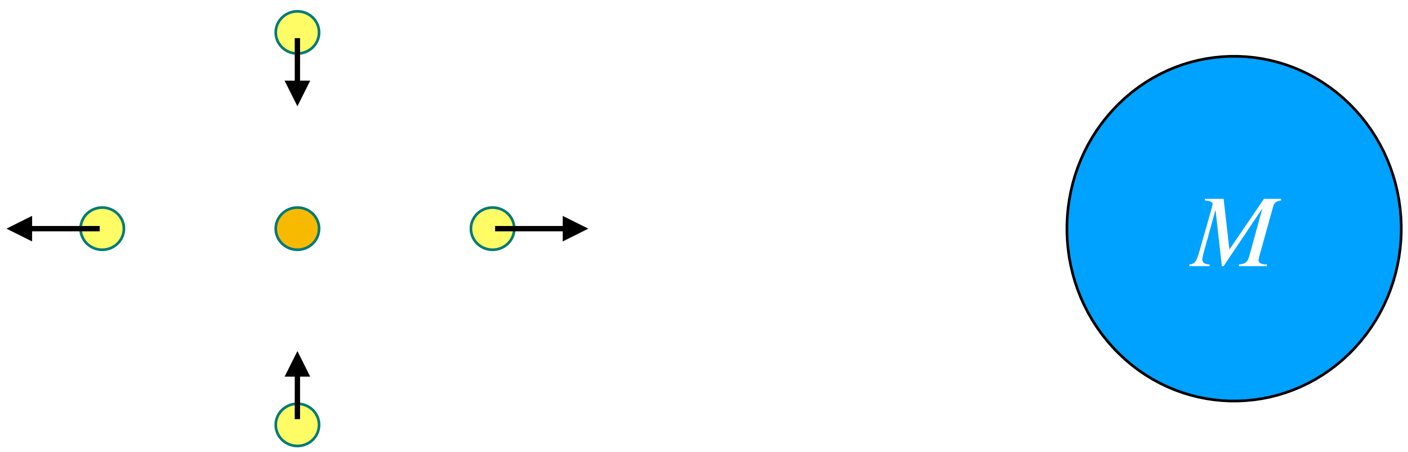
Geodesic deviation equation

Tidal forces

effect of non-uniform gravitational field

appear in the free-falling frame



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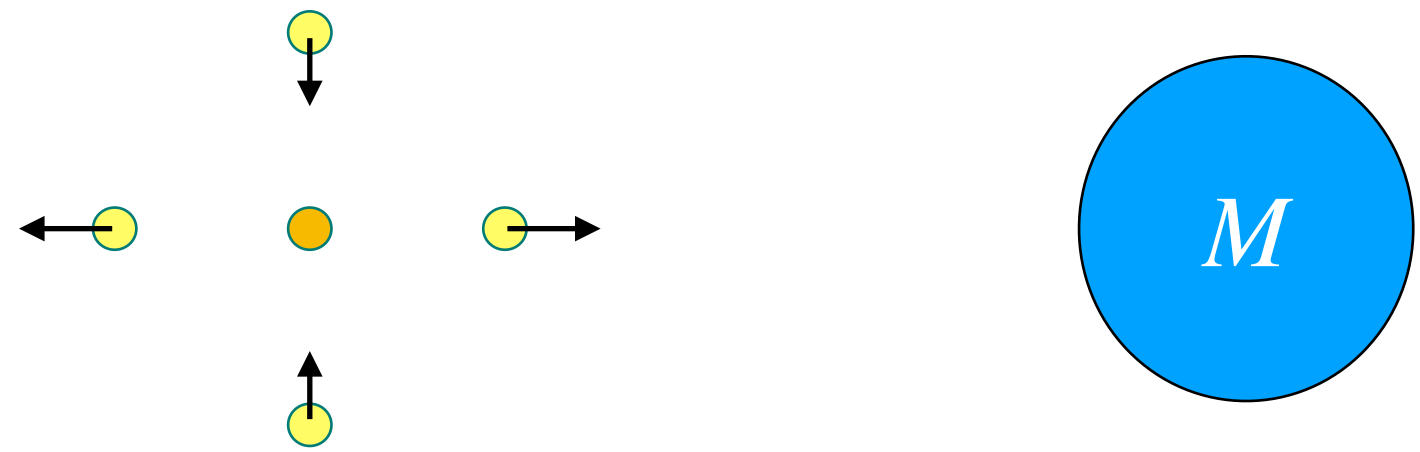

Newtonian approach

$$\ddot{x}_0^i = -\phi_{,i}$$

$$\ddot{x}_0^i + \delta\ddot{x}^i = -\phi_{,i}\left(x_0\right) - \phi_{,ij}\left(x_0\right)\,\delta x^j + O(\delta x^2) \qquad \Longrightarrow \delta\ddot{x}^i = -\phi_{,ij}\left(x_0\right)\,\delta x^j + O(\delta x^2)$$

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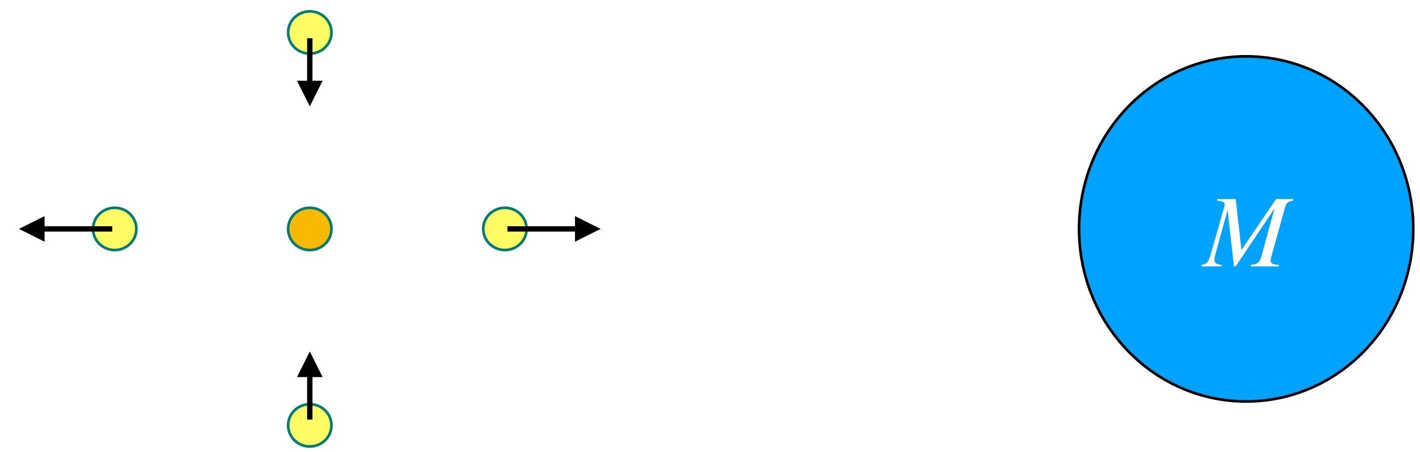

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Tidal tensor

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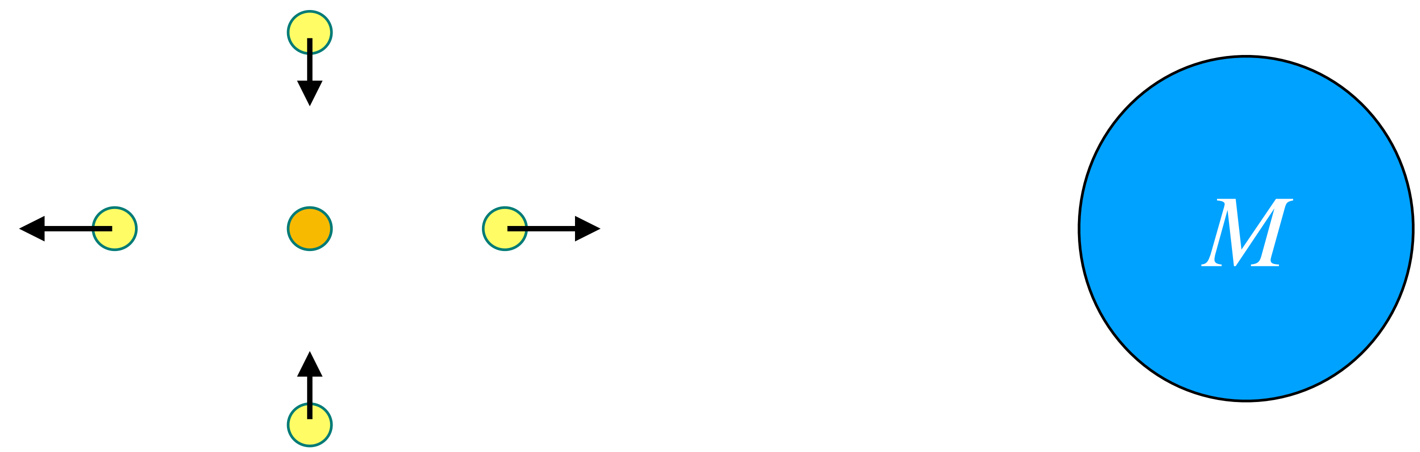

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Tidal tensor

Responsible for tidal deformations, tides etc.

Geodesic deviation equation

GR description

consider a timelike geodesic + slight perturbation

Geodesic deviation equation

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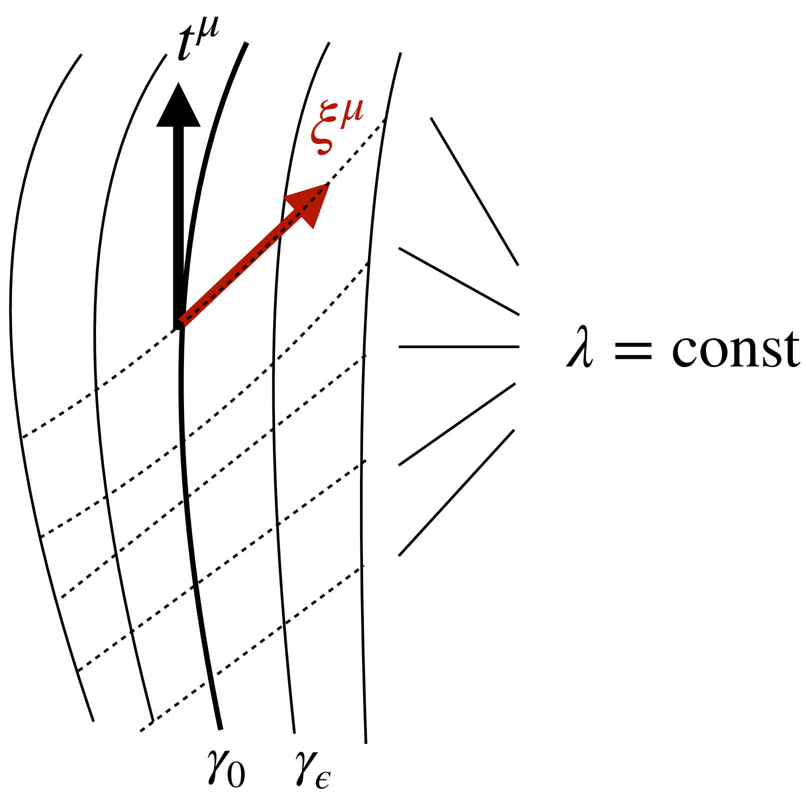
consider a timelike geodesic + slight perturbation

one-parameter family of geodesics $x^\mu(\lambda, \epsilon)$

fiducial geodesic $x^\mu(\lambda,0) \equiv \gamma_0$

tangent vector $t^\mu = \dfrac{\partial x^\mu}{\partial \lambda}$



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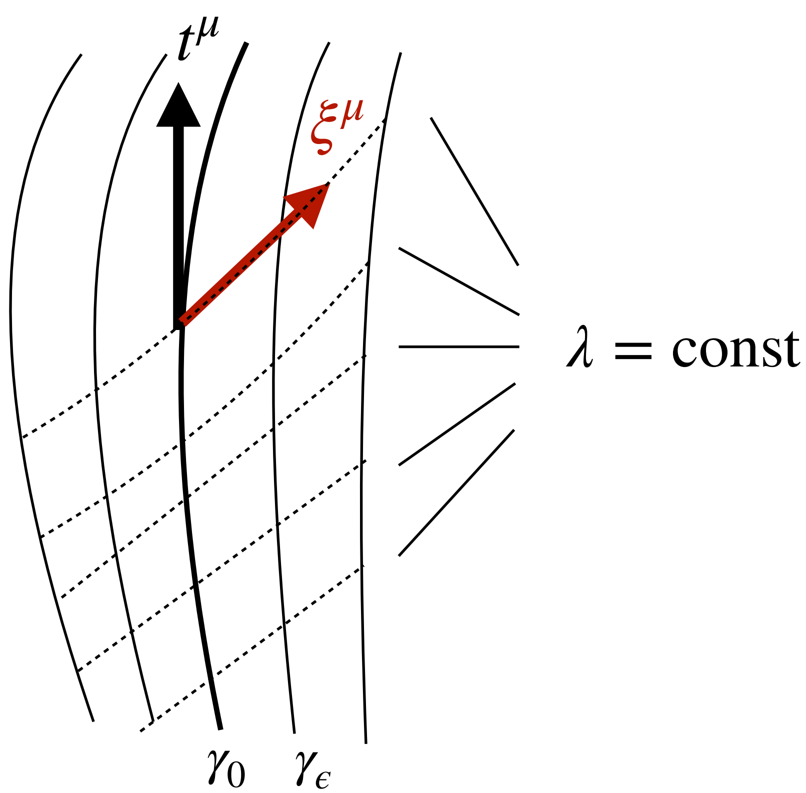

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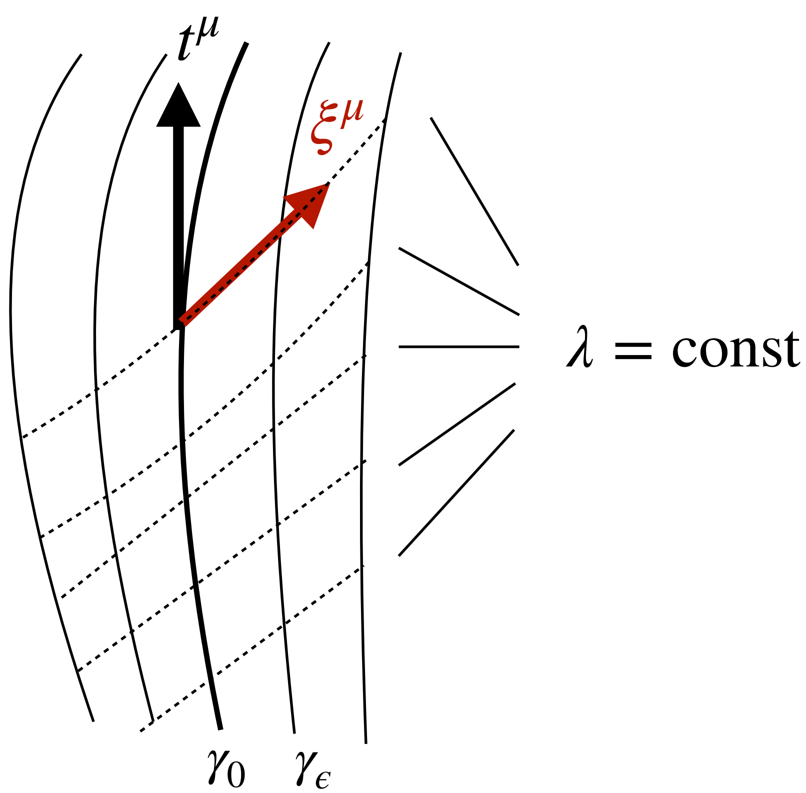

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separation vector
$\xi^\mu(\lambda)$

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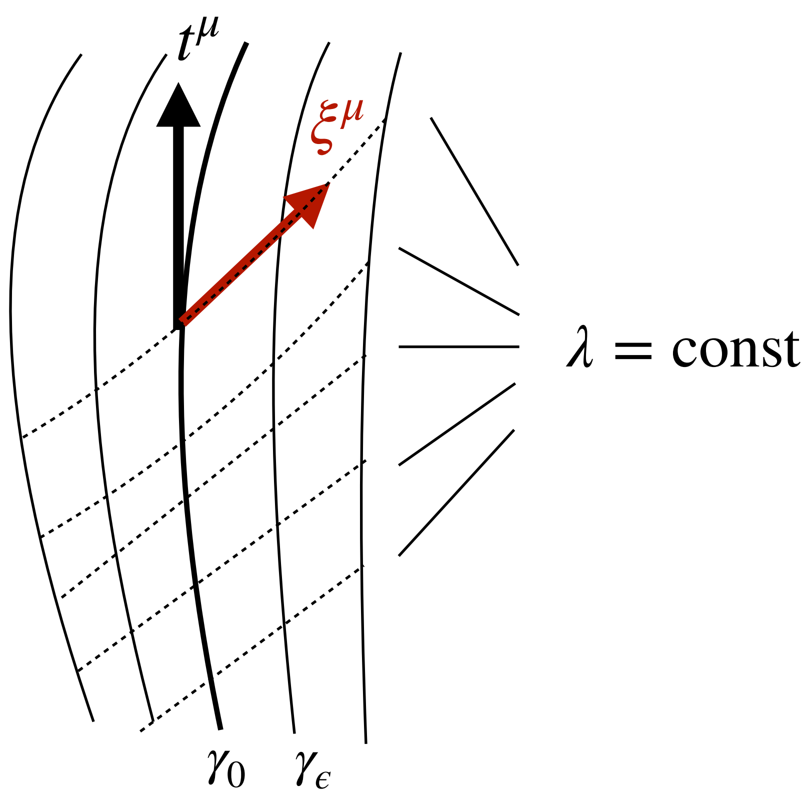

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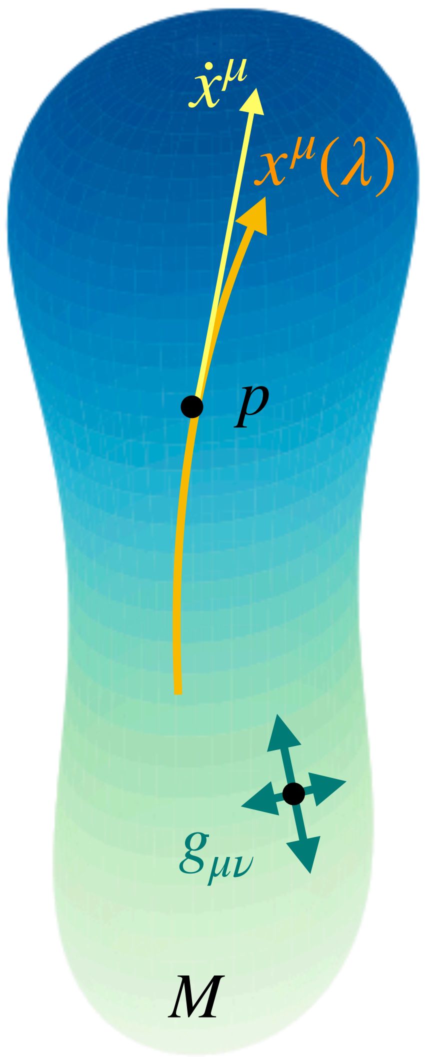
separation vector $\xi^\mu(\lambda)$

geodesic deviation equation $\nabla_t \nabla_t \xi^\mu - R^\mu{}_{\nu\alpha\beta}\, t^\nu\, t^\alpha\, \xi^\beta = 0$

Physical interpretation of the Riemann: governs the tidal forces

Einstein equations

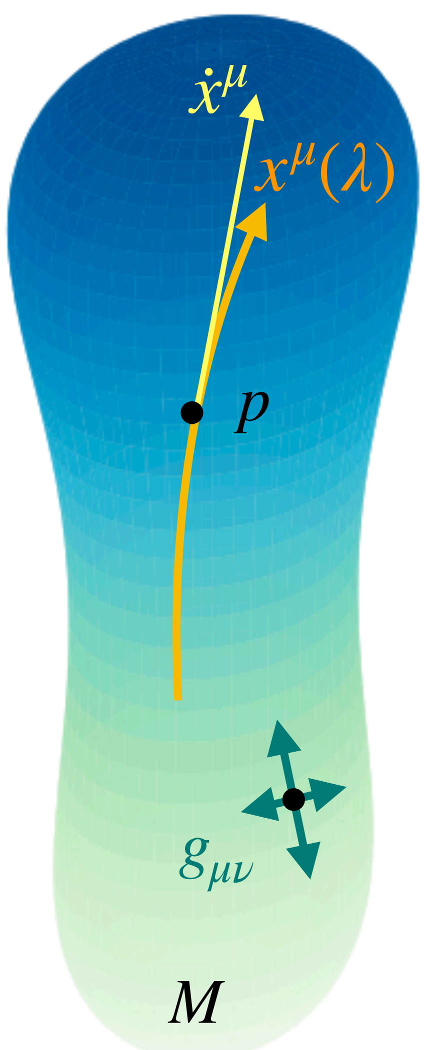
Missing piece of general relativity



1. Spacetime is a manifold (dim = 4) with a Lorentzian metric

Einstein equations

Missing piece of general relativity



1. Spacetime is a manifold (dim = 4) with a Lorentzian metric

2. The metric connection defines locally flat coordinates and local inertial frames

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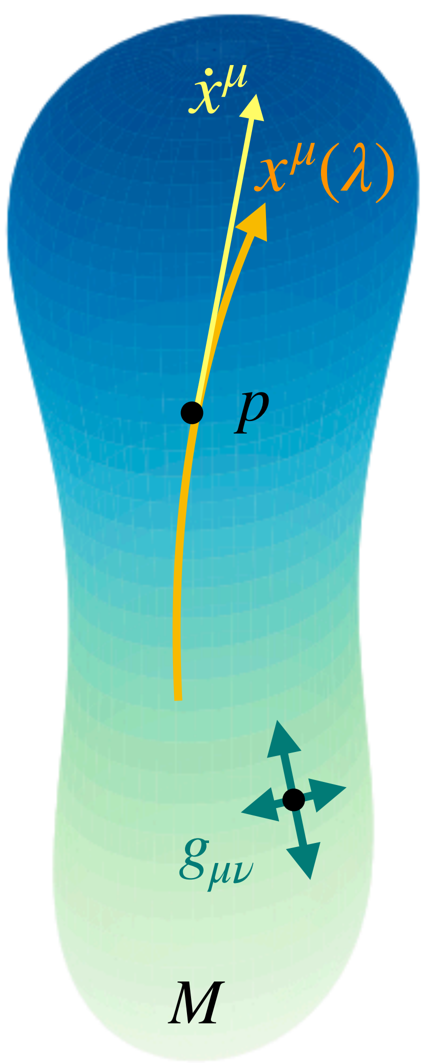

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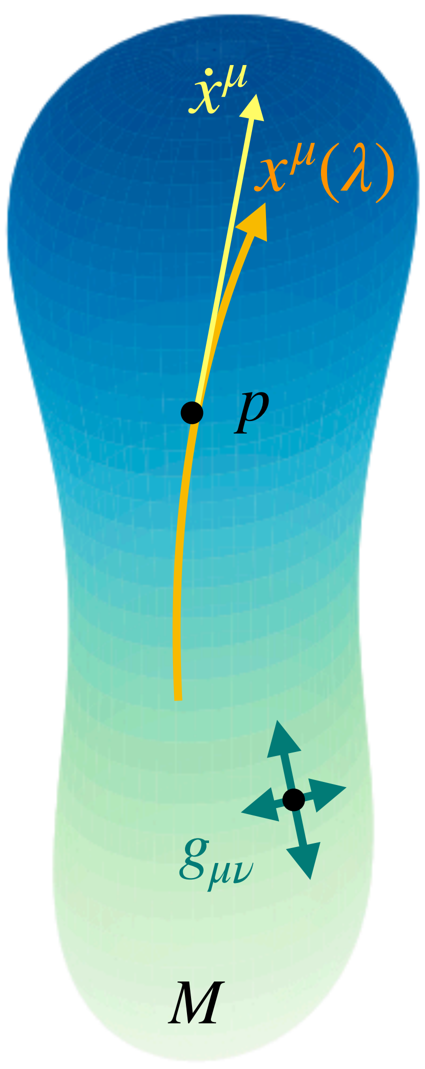

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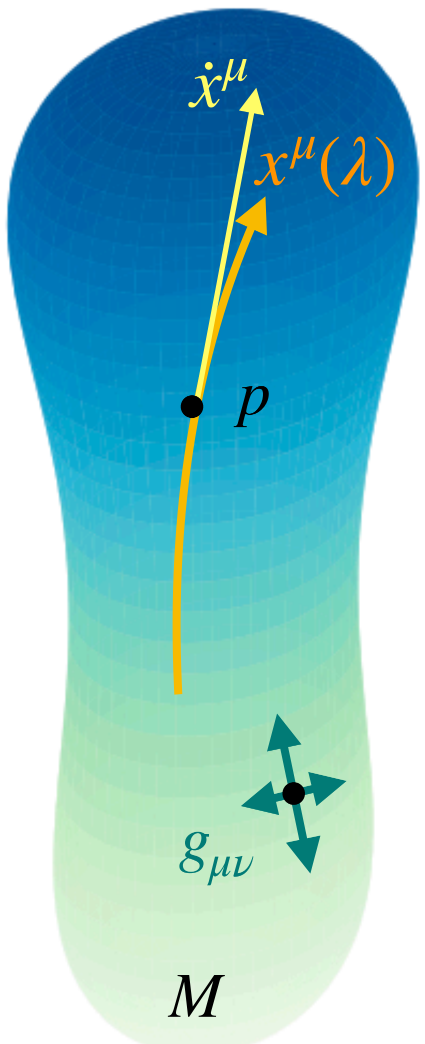

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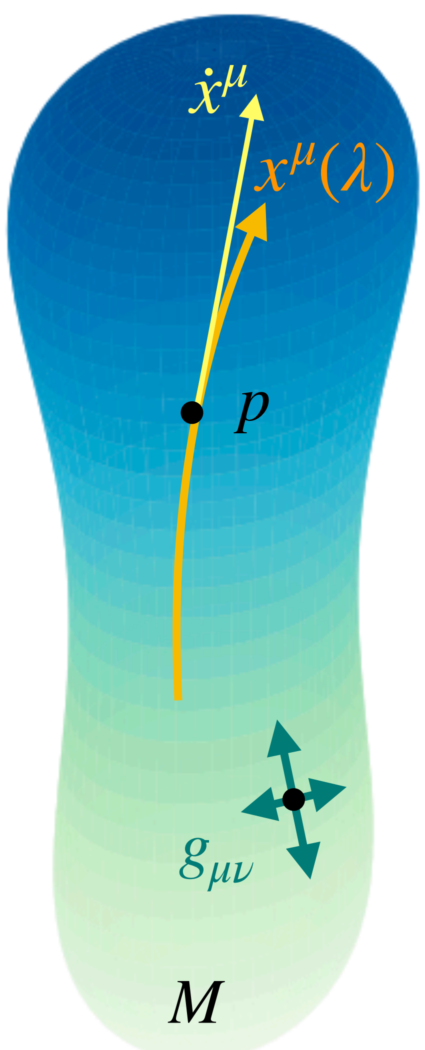

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Idea: Spacetime geometry influences matter, matter should then influence the geometry

Einstein equations

Newtonian theory: we have the Poisson equation

$$\Delta\phi = 4\pi G\,\rho$$

We need its GR counterpart

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What is the counterpart of ϕ, Δ, ρ?

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∂g

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$$\rho = T^{00}$$

Einstein equations

Ideas:

We need field equations for the metric $g_{\mu\nu}(x^\alpha)$ as a field

Metric coupled to matter via the curvature $R^\mu{}_{\nu\alpha\beta}$ and stress-energy tensor $T^{\mu\nu}$

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Maybe $R^{\mu\nu} = \kappa\, T^{\mu\nu}$?

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Bianchi identity $$\nabla_\mu R^{\mu\nu} = \frac{1}{2} R^{,\nu} = 0$$

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Strong restriction on admissible geometries

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How about the Einstein tensor:

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Local energy and momentum conservation built into the theory of gravitation!

Compare the Maxwell's equation and local charge conservation:

$$\partial_\mu F^{\mu\nu} = 4\pi J^\nu \qquad\qquad F^{\mu\nu} = -\,F^{\nu\mu}$$

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…but no additional constraints on the geometry (except the field equations)!

Einstein equations

Consistency with Newtonian theory for small masses and slow motions demands

$$\kappa = 8\pi G$$

Einstein equations:

$$R^{\mu\nu} - \frac{1}{2} R\, g^{\mu\nu} = 8\pi G\, T^{\mu\nu}$$

Remarks:

- if $c \neq 1$ then $R^{\mu\nu} - \frac{1}{2} R\, g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$

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assuming that coordinates $[x^\mu] = \mathrm{m}$

m^{-2}

~~$\mathrm{kg}^{-1} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$~~

$\mathrm{kg}^{-1} \cdot \mathrm{m}^{-1} \cdot \mathrm{s}^{2}$

$\mathrm{J} \cdot \mathrm{m}^{-3}$

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On the other hand:

geometric system of units

$$M_{New} = \frac{G M_{Old}}{c^2} \qquad [\mathrm{m}]$$

$$t_{New} = c\, t_{Old} \qquad [\mathrm{m}]$$

$$\Rightarrow R^{\mu\nu} - \frac{1}{2} R\, g^{\mu\nu} = 8\pi\, T^{\mu\nu}$$

Einstein equations

Einstein equations:

$$R^{\mu\nu} - \frac{1}{2} R \, g^{\mu\nu} = 8\pi G \, T^{\mu\nu}$$

Remarks:

- taking the trace $-R = 8\pi G \, T^{\sigma}{}_{\sigma}$

$$\Rightarrow R^{\mu\nu} = 8\pi G \left(T^{\mu\nu} - \frac{1}{2} T^{\sigma}{}_{\sigma} \, g^{\mu\nu} \right)$$

alternative form of Einstein equations

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in particular: vacuum Einstein equations (no matter)

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- non-linear system of coupled PDE's. Quasi-linear hyperbolic system, initial value problem well-posed (Y. Choquet-Bruhat 1952) given appropriate initial data

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Poisson equation

$$\Delta\phi = 4\pi G\, \rho$$

Cartesian coordinates

$$\phi_{,xx} + \phi_{,yy} + \phi_{,zz} = 4\pi G\, \rho$$

Cylindrical coordinates

$$\phi_{,rr} + \frac{1}{r}, \phi_{,r} + \frac{1}{r^2}\phi_{,\varphi\varphi} + \phi_{,zz} = 4\pi G\, \rho$$

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Einstein equations

$$\Gamma^{\mu}{}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu} \right)$$

$$R^{\mu}{}_{\nu\alpha\beta} = \partial_\alpha \Gamma^{\mu}{}_{\nu\beta} - \partial_\beta \Gamma^{\mu}{}_{\nu\alpha} + \Gamma^{\mu}{}_{\sigma\alpha} \Gamma^{\sigma}{}_{\nu\beta} - \Gamma^{\mu}{}_{\sigma\beta} \Gamma^{\sigma}{}_{\nu\alpha}$$

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Einstein equations

Cosmological constant

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Cosmological constant

- Additional term in Einstein equation involving a constant Λ

Einstein equations
with cosmological constant

$$R^{\mu\nu} - \frac{1}{2} R\, g^{\mu\nu} + \Lambda\, g^{\mu\nu} = 8\pi G\, T^{\mu\nu}$$

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- Later: discovery of accelerated expansion of the Universe (A. Riess et al. 1998, S. Perlmutter et al. 1999) $\Longrightarrow$ Λ needed again

distant supernovae appear dimmer than expected from a cosmological model without cosmological constant

Einstein equations

Fig. 4. from Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant
Riess et al.Vol. 116 1998 AJ 116 1009 doi:10.1086/300499
https://dx.doi.org/10.1086/300499

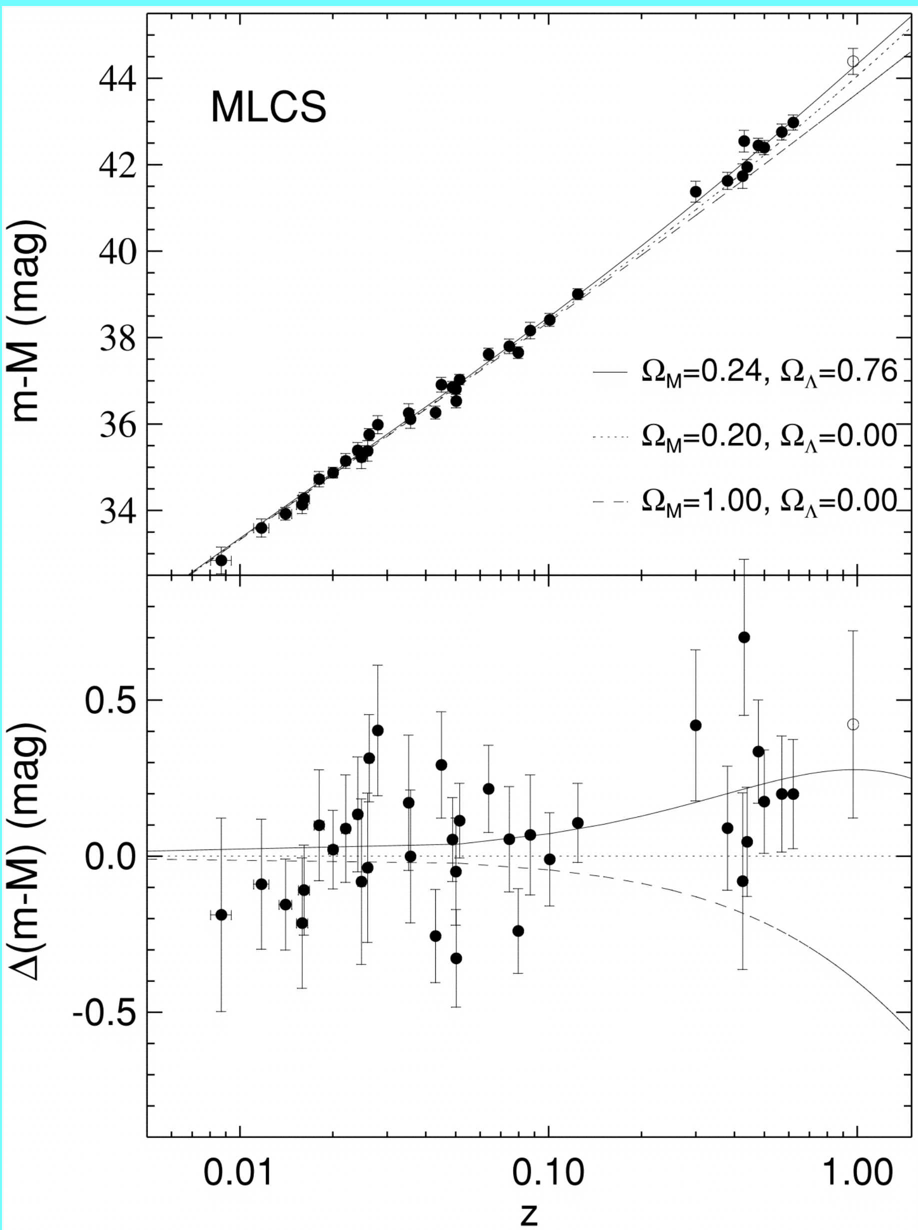
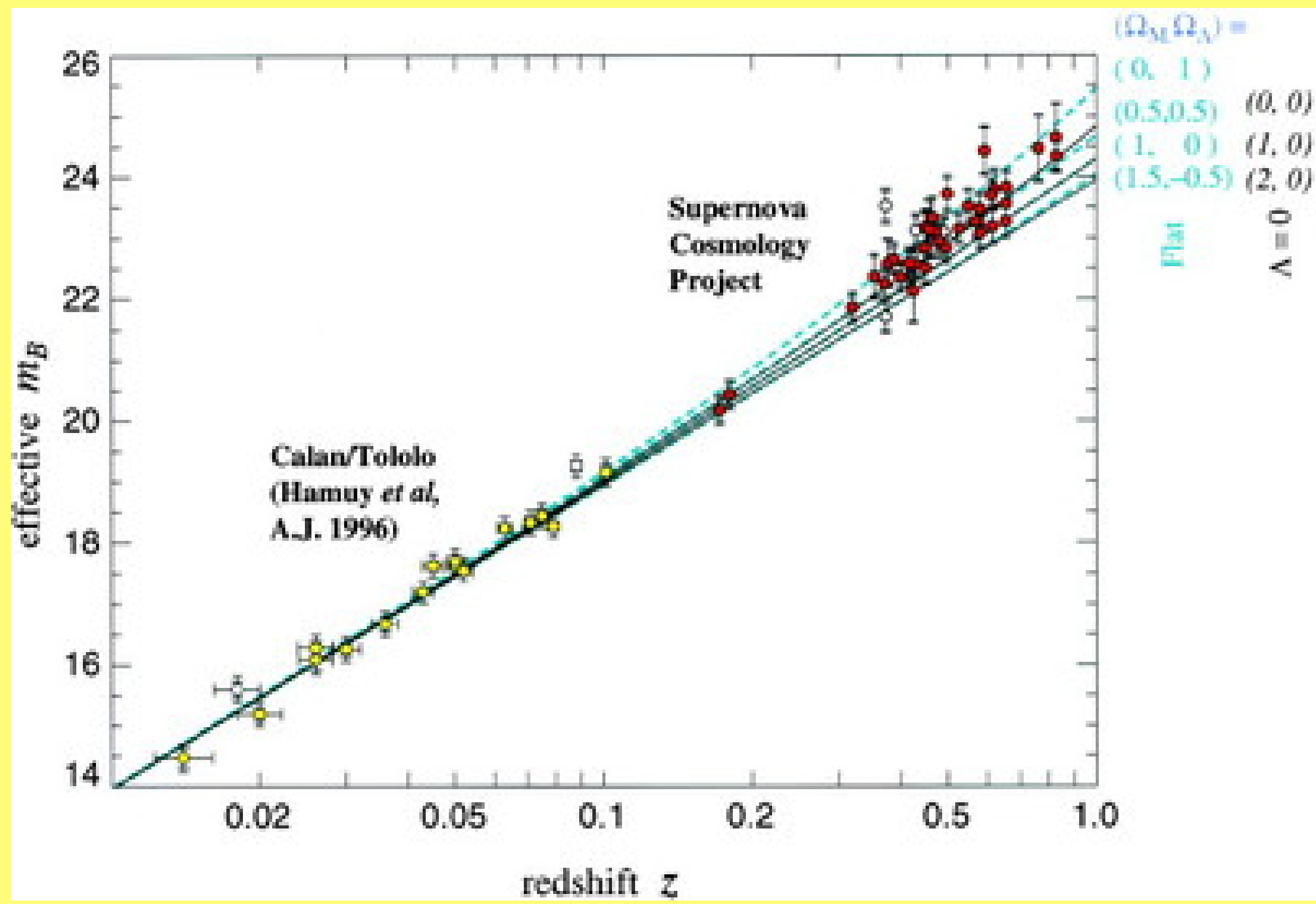


Fig. 1. from Measurements of and from 42 HighRedshift Supernovae
Perlmutter et al. 1999 ApJ 517 565 doi:10.1086/307221
https://dx.doi.org/10.1086/307221



Nobel Prize 2011: S. Perlmutter, B. P. Schmidt, A. Riess

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- $\Lambda = 1.1056 \cdot 10^{-52}\,\mathrm{m}^{-2}$

  very small, becomes relevant only on cosmological scales

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- Gravity acts effectively as a repulsive force on extremely large distances!

- Can be considered a special type of fluid with negative pressure $p = -\rho$

$$R^{\mu\nu} - \frac{1}{2} R\, g^{\mu\nu} = 8\pi G \left(T^{\mu\nu} - \frac{\Lambda}{8\pi G}\, g^{\mu\nu} \right)$$

energy density $5.3 \cdot 10^{-10}\,\mathrm{J\,m^{-3}}$

mass density $5.9 \cdot 10^{-27}\,\mathrm{kg\,m^{-3}}$

Einstein equations

Variational principle for the Einstein equations

$$S_\phi = \int \mathscr{L}(\phi, \phi_{,i})\, d^4x \qquad\qquad \delta S_\phi = 0\,(+\text{boundary terms}) \Longrightarrow \text{field equations for } \phi$$

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  - useful for quantizing gravity

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- The variational principle must be coordinate-system invariant

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scalar density

$$F \mapsto F' = F\cdot\left|\det \Lambda^\mu{}_{\nu'}\right| = F\cdot\left|\det \Lambda^{\nu'}{}_\mu\right|^{-1}$$

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Hilbert's variational principle

- Now $\hat{\mathscr{L}}(g_{\alpha\beta}, \partial_\mu g_{\alpha\beta}, \partial_\mu \partial_\nu g_{\alpha\beta}, \ldots)$ needs to be a scalar

  How about the Ricci scalar $R \equiv R(g_{\alpha\beta}, \partial_\mu g_{\alpha\beta}, \partial_\mu \partial_\nu g_{\alpha\beta})$?

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$$S = S_g + S_m$$

$$S_g = \frac{1}{16\pi G} \int R \sqrt{-g}\, d^4x \qquad\qquad S_m = \int \hat{\mathscr{L}}_m(\Phi, \nabla_\mu \Phi, g_{\alpha\beta}) \sqrt{-g}\, d^4x$$

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variation wrt to the metric

$$\delta S = \int \frac{1}{16\pi G} \left(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \right) \delta g^{\mu\nu} \sqrt{-g}\, d^4x + \int \frac{\delta S_m}{\delta g^{\mu\nu}} \delta g^{\mu\nu}\, d^4x$$

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- Palatini's approach: the same Lagrangian, but $R \equiv R(\Gamma^\mu{}_{\alpha\beta}, \partial_\gamma \Gamma^\mu{}_{\alpha\beta}, g_{\alpha\beta})$

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  - …

End of lecture 7

Linearized Einstein equations

Assumption

There exists a coordinate system (x^μ) in which $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x^\sigma)$

const
„flat background"

$|h_{\mu\nu}| \ll 1, \quad |h_{\mu\nu,\alpha}| \ll 1$
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The spacetime appears to be almost flat over large regions (except the immediate vicinity of massive compact objects)

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Propagation of gravitational waves over large distances best described by linearized equantions

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First goal: impose the Einstein equations on $h_{\mu\nu}(x^\sigma)$, find the simplest form of the resulting equations